

Math 451

Proj. 6 → on blackboard.

Dijk.m

→ write centrality.m

← given
(see previous lectures)

Number theory

Set: integers

ops: $+$, $-$, $*$

division algorithm
 div , mod

↑
int64
uint64

div-mod as floats

$$a = \overset{\text{div}}{q} \cdot d + \overset{\text{mod}}{r}$$

$$q = \lfloor a/d \rfloor$$

$$r = a - q \cdot d$$

div-mod only $+$, $-$

function $[q, r] = \text{div-mod}(a, d)$

$$q = 0;$$

$$r = a;$$

while $r \geq d$

$$q = q + 1;$$

$$r = r - d;$$

end

end

is positive

function [q, r] = div-mod(a, d) ^{pos or neg}

$$q = 0;$$

$$r = \text{abs}(a);$$

while r >= d

$$q = q + 1;$$

$$r = r - d;$$

end

if a < 0

$$q = -(q + 1);$$

$$r = d - r;$$

end

end

$$-6 = (-2)4 + 2$$

$$6 = (1)4 + 2$$

$$-7 = (-3)3 + 2$$

$$7 = (2)3 + 1$$

$$\begin{array}{r} -7 \quad 0 \quad 7 \\ \hline 10 \quad 10 \quad 10 \\ \hline 3 \quad 3 \quad 3 \quad 3 \quad 3 \end{array}$$

Mod function:

$$\text{mod}(a + b, m) = \text{mod}(\text{mod}(a, m) + \text{mod}(b, m), m)$$

$$\text{mod}(a \cdot b, m) = \text{mod}(\text{mod}(a, m) \cdot \text{mod}(b, m), m)$$

base b expansions

$$(1, 2, 3)_{10} = 1 \cdot 10^2 + 2 \cdot 10^1 + 3 \cdot 10^0$$

$$(1, 0, 0, 1)_2 = 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

$$(2, 3, 0, 4)_7 = 2 \cdot 7^3 + 3 \cdot 7^2 + 0 \cdot 7^1 + 4 \cdot 7^0$$

$$(a_2, a_1, a_0)_b = a_2 \cdot b^2 + a_1 \cdot b + a_0$$

$$n = (a_2, a_1, a_0)_b = a_2 \cdot b^2 + a_1 \cdot b + a_0$$

$$n = a_2 b^2 + a_1 b + a_0$$

$$n = (a_2 b + a_1) \cdot b + a_0$$

$$n = q \cdot b + r$$

$$\text{div_mod}(n, b)$$

$$q = a_2 b + a_1$$

$$r = a_0$$

$$n_2 = a_2 b + a_1$$

$$\text{div_mod}(n_2, b)$$

$$q = a_2 \rightarrow n_3$$

$$r = a_1$$

$$n_3 = a_2 \neq 0$$

function $\Sigma a = \text{baseb}(n, b)$

$$q = n;$$

$$k = 1;$$

while $q \neq 0$

$$[q, r] = \text{div_mod}(q, b)$$

$$a[k] = r;$$

$$q = q - r \cdot b;$$

$$k = k + 1;$$

end

$$a = \text{flip}(a);$$

end

engine of R.S.A.

$$a^b \pmod{m}$$

$$\rightarrow \underbrace{(a \cdot a \cdot a \cdot \dots \cdot a)}_{b\text{-times}} \pmod{m}$$

function $\{pm\} = \text{power}(a, b, m)$

$$pm = 1;$$

for $i = 1 : b$

$$pm = pm \pmod{m} (a * pm, m);$$

end

Base b

$$(a_k b^{k-1} + a_{k-1} b^{k-2} + \dots + a_1 b + a_0)$$

$$((2)_3 + 0)_3 + 2$$

$$\left(\underbrace{(0 \cdot 3 + 2)}_3 \cdot 3 + \underbrace{10}_3 \right)_3 + 2$$