Math 322

$\boxed{\text{Q's}}$ $\boxed{9.1 \ \#5}$ $R = \{ (a,b) \mid a,b$ are web pages and everyone who visits a visits $b \}$

T a) ref. $\forall a \ aRa \equiv \forall a$ "everyone who visits a, visits a"

F b) irreflexive $\forall a \ a \not R a \equiv \forall a$ "everyone who visits a, doesn't visit a"

counterexample "chaos R chaos"

F c) sym $\forall a \forall b \ (aRb \to bRa)$

$\equiv$ see video for discussion

d) anti sym

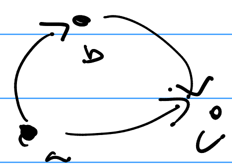
e) asym

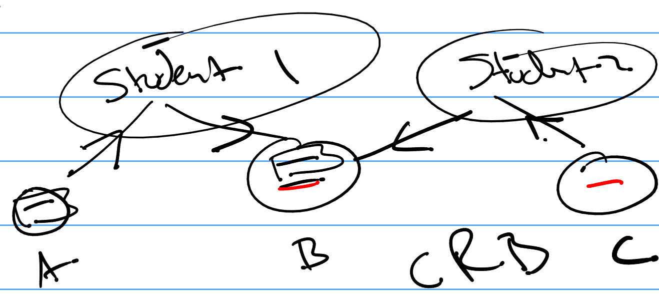
f) trans $\forall a \forall b \forall c \ (aRb \wedge bRc \to aRc)$

$\equiv$

see $\boxed{\text{discussion}}$ of video

$\boxed{\text{True}}$





CRB

ARB

R everyone who has visited □ $\boxed{\text{has also}}$ visited △

is this implx? Yes

□ has also △

visited □ w. △

$\boxed{9.3}$ Represent R on Set A

1. lists
2. set builder notation
3. Zero-One Matrix $\quad M_R = [m_{ij}]$

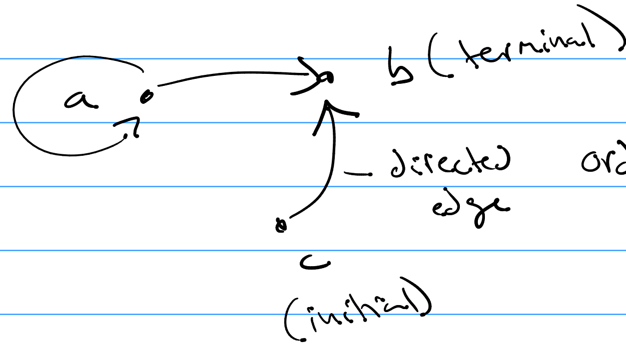
$$m_{ij} = \begin{cases} 1 & a_i R b_j \\ 0 & a_i \not R b_j \end{cases}$$

$R = \{(a,b), (a,c), (c,b)\} \qquad A = \{a,b,c\}$

&

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

4. Digraphs



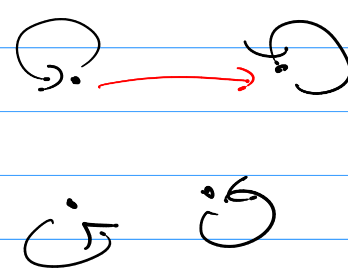
ordered pair = connecting edge

Why?

1. Properties are easy to see (except trans)

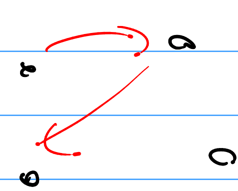
reflexive:

$$\begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix}$$

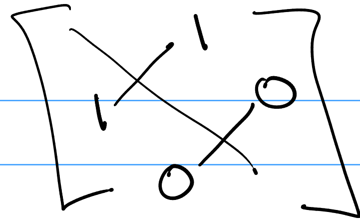


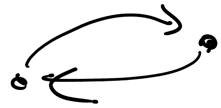
irreflexive

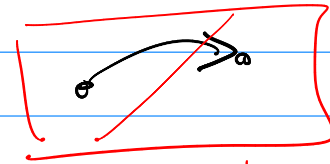
$$\begin{bmatrix} 0 & & \\ & \ddots & \\ & & 0 \end{bmatrix}$$



symm

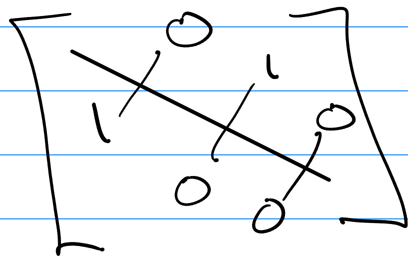


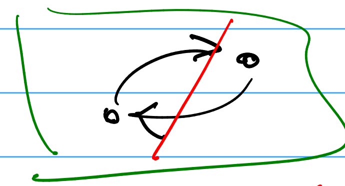




counterexample

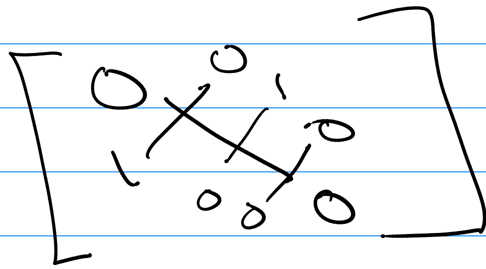
antisym





counterexample

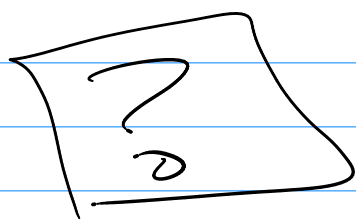
asym

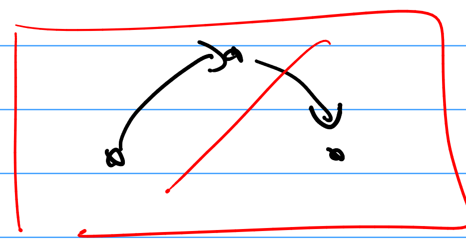


no loops

no sym. edges

trans





counterexample

Zero-One Matrices have computational ops

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2}$$

$$M_{R_1 \circ R_2} = M_{R_2} \odot M_{R_1}$$

$$M_{R^n} = M_R^{[n]}$$

Having relations that are $\boxed{\text{reflexive}}$ and/or $\boxed{\text{sym}}$ and/or $\boxed{\text{anti sym}}$ and/or (any property)

can be useful.