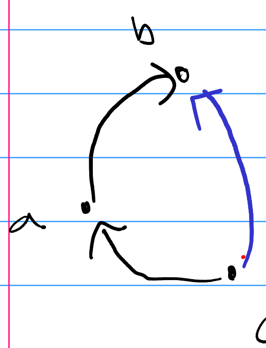
Math 322

Transitive Closure R on A where $|A| = n$

$$M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]} \vee \cdots \vee M_R^{[n]}$$

Warshall's Algorithm



$$M_R = \begin{array}{c|ccc} & a & b & c \\ a & 0 & 1 & 0 \\ b & 0 & 0 & 0 \\ c & 1 & 1 & 0 \end{array}$$

← a goes to ...

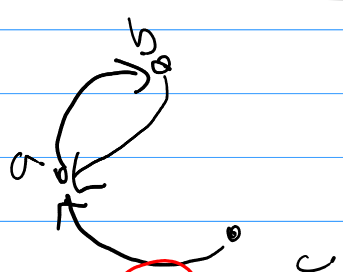
who goes to a

Idea: check for transitive by focusing on ...

1. a in the middle ← Warshall's first step
2. b in the middle
3. c in the middle

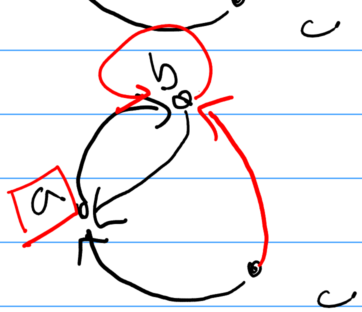
⋮

Ex



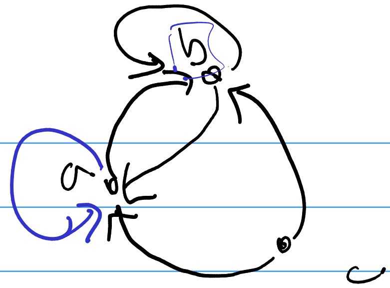
$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

1st step



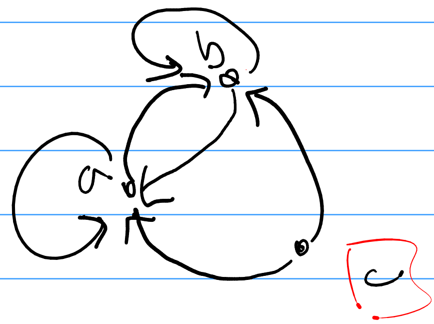
$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

2^{nd} step



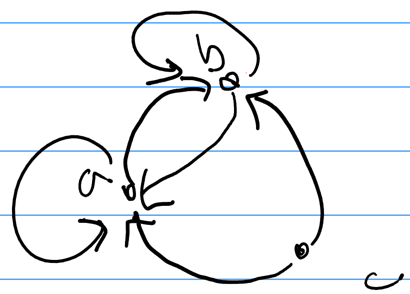
$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

3^{rd} step



$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

4^{th} step



$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

R^* $\qquad\qquad$ M_{R^*}

$$M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$M_{R^*} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

watch video!

9.5 if R on set A is reflexive, symm, and transitive, we call R an equivalence relation.

Notation: when R is an equiv. relation

$(a,b) \in R$, aRb

use $\boxed{a \sim b}$ → "a is equiv. to b"

$\sim = \{(a,b) \mid$ a is related to b by blah $\}$

ref, sym, tran

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ex $R = \{(a,b) \mid \boxed{a \equiv_m b}\}$

$3 \equiv_4 -1 \equiv_4 -9$

$3 \equiv_4 7 \equiv_4 11 \equiv_4 15$

① $m \mid a-b$

② $a = b + mk$

③ $a \bmod m = b \bmod m$

Is $a \equiv_m b$ ref? sym? trans?

Y Y Y

So R $\underline{\underline{\text{is}}}$ an equiv. relation.

Def equiv. class

$$[a]_R = \{ e \mid a \sim e \}$$

ex

$$R = \{ (a,b) \mid a \equiv_4 b \}$$

$$\boxed{\begin{aligned} [0]_R &= \{ \dots, -8, -4, 0, 4, 8, \dots \} \\ [1]_R &= \{ \dots, -7, -3, 1, 5, 9, \dots \} \\ [2]_R &= \{ \dots, -6, -2, 2, 6, 10, \dots \} \\ [3]_R &= \{ \dots, -5, -1, 3, 7, 11, \dots \} \end{aligned}} = \mathbb{Z}$$

partition of $\mathbb{Z}$

$$[4]_R = \{ \dots -4, 0, 4, 8, \dots \}$$

Thm these are logically equivalent

① $a \sim b$

② $[a] = [b]$

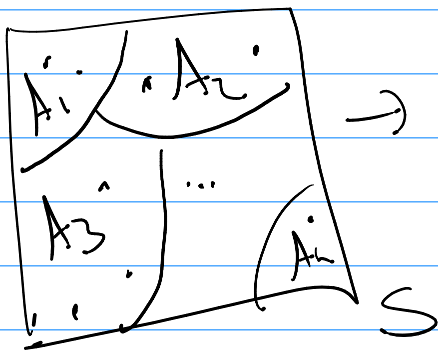
③ $[a] \cap [b] \neq \emptyset$

Partition

A_i are a partition of set S

if

1. $A_i \neq \emptyset$
2. $A_i \cap A_j = \emptyset$ if $i \neq j$
3. $\bigcup_i A_i = S$



→ Say A_i are a partition of S

Thm R is an equiv relation on S

→ Equiv Classes form a partition of S

And given any partition of S, A_i

then there exists an equiv. relation

such that its equiv classes $= A_i$.