Math 322

th^m Principle of Well ordered Induction

$$\left[P(1^{st}) \wedge \left\{\forall k\left(P(1^{st}) \wedge P(2^{nd}) \wedge \cdots \wedge P(k^{th})\right) \rightarrow P(k+1^{st})\right\}\right] \rightarrow \forall n P(n)$$

T   T $\wedge$ F

Pf: (use contradiction)

assume: $P(1^{st}$ case$)$ is true

assume: Induction Step is true

$$P(1^{st}) \wedge P(2^{nd}) \wedge \cdots \wedge P(k^{th}) \rightarrow P(k+1^{st})$$

$$\begin{array}{ccccc} T & T & \cdots & T & T \end{array}$$

assume: $\forall n P(n)$ is false

$\rightarrow$ $P(n)$ has one or more counter examples

b/c our set is well ordered, our counter examples can be seen as $\boxed{1^{st} \text{ counter example}}$, 2^{nd} c.e, ...

So

$P(n)$ $P(1)$, $P(2)$, .. , $P(\ )$, $\boxed{P(1^{st} \text{ counter ex.})}$, ..

$$\begin{array}{cccccc} T & T & \cdots & T & F & \nabla \end{array}$$

Contradiction

but the inductive step says.. $P(1^{st}$ c.e$) \equiv T$

$\emptyset$

$0,\ 1,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \frac{2}{3},\ \frac{1}{5}$

$$\begin{array}{cccc}
\frac{1}{1} & \frac{2}{1} & \frac{3}{1} & \frac{4}{1} \cdots \\
\frac{1}{2} & \frac{2}{2} & \frac{3}{2} & \frac{4}{2} \cdots \\
\frac{1}{3} & \frac{2}{3} & \frac{3}{3} & \frac{4}{3} \cdots \\
\frac{1}{4} & 2/4 & 3/4 & 4/4 \cdots \\
\frac{1}{5} & 2/5 & & \\
\frac{1}{6} & & &
\end{array}$$

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lexicographic order (dictionary sort)

$n < m$ $\quad a = (a_1, a_2, \ldots, a_n) \quad b = (b_1, b_2, \ldots, b_m)$

$\boxed{a < b}$ when $a_1 < b_1$

or if $a_1 = b_1$, $a_2 < b_2$

or if $a_2 = b_2$, $a_3 < b_3$

$\vdots$

$a_{n-1} = b_{n-1}$, $a_n < b_n$

or if $a_n = b_n$ but $m > n$

Hasse Diagrams

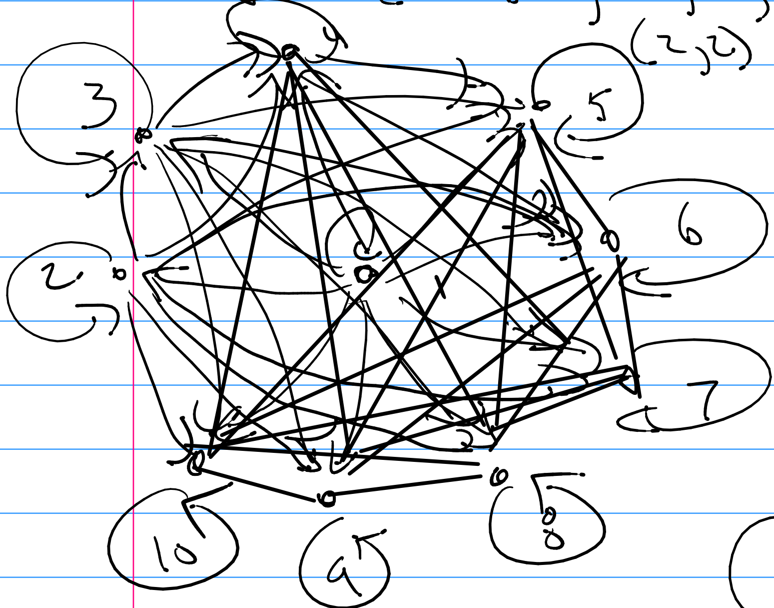
$(\{1,2,\dots,10\}, \leq)$

$R = \{(1,1),(1,2),(1,3),\dots,(1,10)$

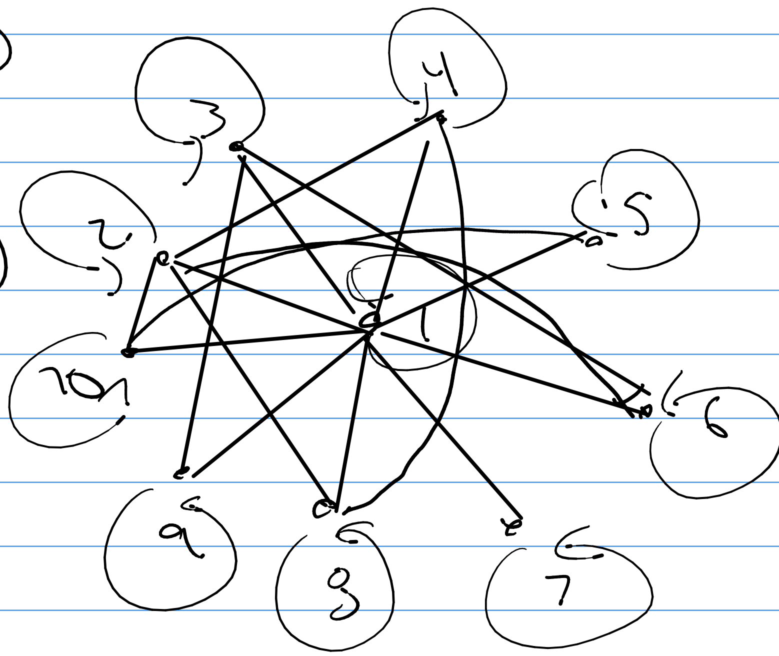
$(2,2),(2,3),\dots,(2,10)$

$\vdots$

$(10,10)\}$



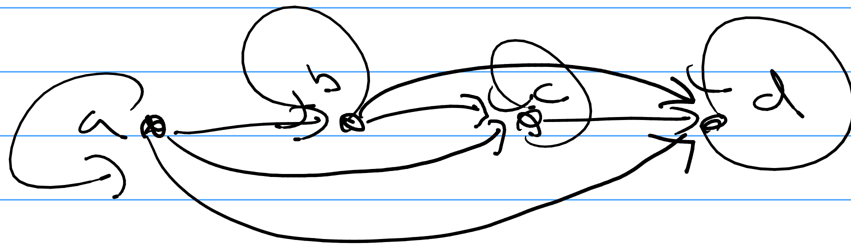
$(\{1,2,\dots,10\}, |)$

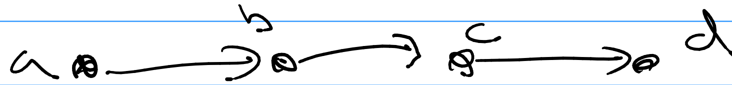


Simplify the graph so the ref, antisym, trans are assumed (b/c (5,5)) but not specifically drawn

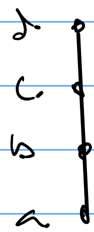
1. ref. (do not draw loops)

2. trans $\equiv \forall n \; R^n \subseteq R$



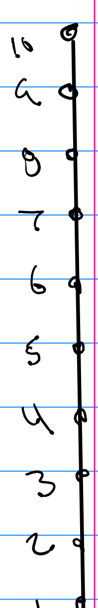


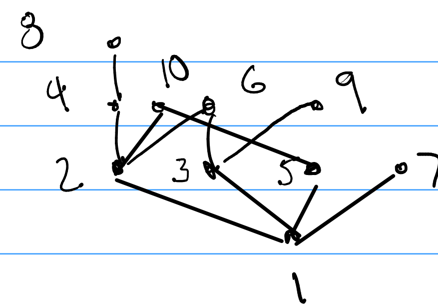
3. antisym. (remove arrows and assume arrows point up)



$\boxed{\text{Hasse Diagram}}$

$(\{1, 2, \ldots, 10\}, \leq)$ $\quad$ $(\{1, 2, 3, \ldots, 10\}, \mid)$





$\boxed{\text{Extremal Properties of Hasse Diagrams}}$