Math 322

$\boxed{Q 1 b}$ $R = \{ (a,b) \mid a \neq b \}$ on integers

2 → aRb is really all reals except $a \not R a$

$$\boxed{\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}} \vee \boxed{\begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$\boxed{R = \mathbb{Z} \times \mathbb{Z}}$

$\boxed{R \cup \Delta}$ ← ref. closure

3 Sym. closure $M_R \vee M_R^T$

$\boxed{R \cup R^{-1}}$

$R = \{ (a,b) \mid a|b \}$

$R^{-1} = \{ (a,b) \mid b|a \}$

$$R \cup R^{-1} = \{ (a,b) \mid a|b \text{ or } b|a \}$$

9.1 example $R = \{ (a,b) \mid a|b \vee b|a \}$

Ref?

Iref?

Sym?

anti sym?

asym?

trans?

✓ (1) Reflexive $\boxed{\forall a R a} \equiv$ "$a|a$ or $a|a$" $\boxed{\text{True}}$

(2) Irreflexive $\forall a \not R a \equiv$ "$a \nmid a \wedge a \nmid a$"

False: $2|2$ counterexample

✓ (3) Sym $\forall a \forall b (aRb \to bRa)$

$\equiv [a|b \vee b|a \to b|a \vee a|b]$

if $a|b \vee b|a$ is True $\to$ $b|a \vee a|b$ must be true as well

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4) Asym $\forall a \forall b (aRb \to b \not R a)$

(Know Asym $\equiv$ irref $\wedge$ antisym)

5) Antisym $\forall a \forall b (aRb \wedge bRa \to a = b)$

$\equiv \forall a \forall b (a \neq b \to \neg(aRb \wedge bRa))$

$\equiv \forall a \forall b (a \not R b \vee b \not R a \vee a = b)$

$\equiv \neg \exists a \exists b (aRb \wedge bRa \wedge a \neq b)$

6) Trans $\forall a \forall b \forall c (\boxed{aRb} \wedge bRc \to aRc)$

$\equiv [(a|b \vee b|a) \wedge (b|c \vee c|b) \to (a|c \vee c|a)]$

3|6 2|6 3∤2 2∤3

$\boxed{3R6}$ 6R3 2R6 $\boxed{6R2}$

$R = \{ (a,b) \mid$ a is a factor of b or a is a mult. of b $\}$

$2R2$ $\quad 2R4 \quad \boxed{2R6}$ $(2,6)$

$\boxed{6R2}$ $(6,2)$

Not trans

$2R6 \wedge 6R3 \rightarrow 2 \cancel{R} 3$

counter example

Exam 1 13 probs @ 10 pts each

120 pts = 100%

$\boxed{9.1}$ (3 probs)

① Check all 6 properties (see above)

$R = \{(a,b) \mid$ a hit b with a bat$\}$

$A =$ all teachers @ WSU

Ref: $\forall a\,(aRa) \equiv$ "a hit a with a bat"

True? False? $\rightarrow$ counter example

reasonable statement

② $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \oplus R_2$

given $R_1 = \{ \quad \}$

$R_2 = \{ \quad \}$

③ <u>Prove:</u> If R is trans $\rightarrow \forall n \; R^n \subseteq R$, $n = 1, 2, 3 \ldots$

pf (uses induction)

9.2 (1 prob)

① Composite Key

9.3 (2 probs)

① Set $\rightleftarrows$ digraph $\rightleftarrows$ matrix

② $M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2}$

$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2}$

$M_{R_1 \circ R_2} = M_{R_2} \odot M_{R_1}$

9.4 (3 probs)

① a) Sym Closure

b) Ref. Closure

② R^* using $M_R \vee M_R^{[2]} \vee \cdots \vee M_R^{[n]}$

③ R^* using <u>warshall's</u>

9.5 (2 probs)

① Is R an equiv. relation?

② Find $[a]_R$

9.6 (2 probs)

① Is (S, R) a poset?

② given Hasse diagram

a) Extremals

b) topological Sort (left side)
