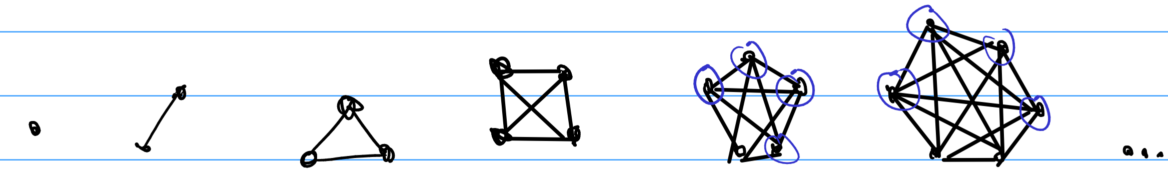
# Math 322

## 10.2 Special Simple Graph

Complete : $K_n$ has $n$-vertices and edge between all vertices.



| | $K_1$ | $K_2$ | $K_3$ | $K_4$ | $K_5$ | $K_6$ | $K_n$ |
|---|---|---|---|---|---|---|---|
| Vertices | 1 | 2 | 3 | 4 | 5 | $\boxed{6}$ | $n$ |
| Edges | 0 | 1 | 3 | 6 | $4+3+2+1$ <br> 10 | $5+4+3+2+1$ <br> 15 | $\frac{(n-1)n}{2}$ |
| $\deg(v_i)$ | 0 | 1 | 2 | 3 | 4 | 5 | $n-1$ |

Handshake thm $\sum \deg(v) = 2\lvert E \rvert$

$$n(n-1) = \cancel{2}\,\frac{(n-1)(n)}{\cancel{2}}$$

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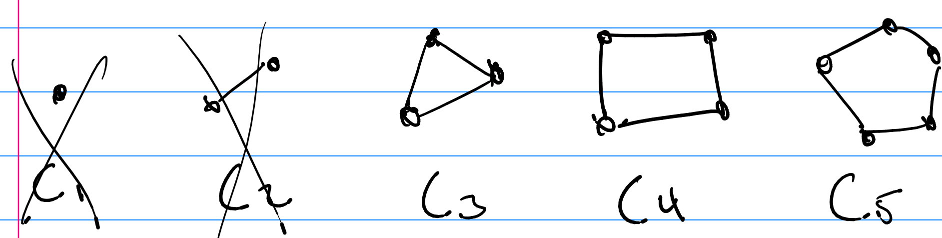
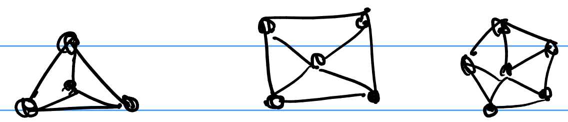
Cycle : $C_n$ $n$ - Vertices and edges are
$\rightarrow \{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_{n-1}, v_n\}, \{v_n, v_1\}$



| | ~~$C_1$~~ | ~~$C_2$~~ | $C_3$ | $C_4$ | $C_5$ | $\ldots$ | $C_n$ |
|---|---|---|---|---|---|---|---|
| Vertices | | | 3 | 4 | 5 | | $n$ |
| edges | | | 3 | 4 | 5 | | $n$ |
| $\deg(v)$ | | | 2 | 2 | 2 | | 2 |

wheel : $W_n = C_n$ + one extra vertex
that connects to each
n-vertices in $C_n$



| | $W_3$ | $W_4$ | $W_5$ | ... | $W_n$ |
|---|---|---|---|---|---|
| vertices | 4 | 5 | 6 | | $n+1$ |
| edges | 6 | 8 | 10 | | $2n$ |
| deg(v) | $C_3 = 3$ | $C_4 = 3$ | $C_5 = 3$ | | $C_n = 3$ |
| | axel $= 3$ | axel $= 4$ | axel $= 5$ | | axel $= n$ |

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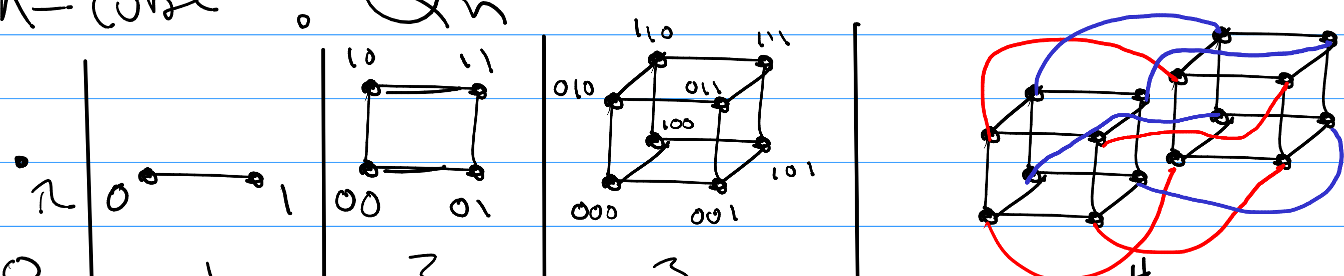
$n^{th}$ dimensional Cubes

n-cube : $Q_n$




| n | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| vertices | 1 | 2 | 4 | 8 | 16 |
| edges | 0 | 1 | 4 | $12 = 3 \cdot 2^2$ | $32 = 4 \cdot 2^3$ |
| deg | 0 | 1 | 2 | 3 | 4 |

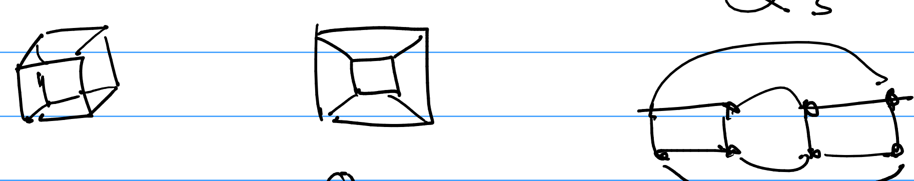
$Q_n$ vertices $= 2^n$

edges $= \frac{1}{2} n 2^n = n 2^{n-1}$

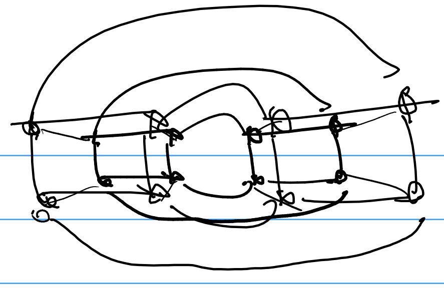
$\deg(x) = n$

$Q_3$

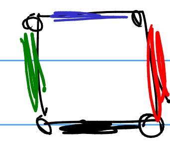
$Q_3$



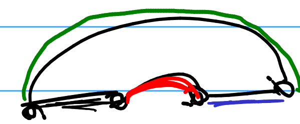
$Q_3$



$Q_4$



$Q_2$



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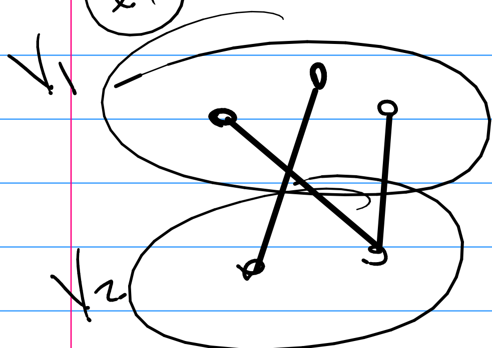
## Bipartite Graph

$G = (V, E)$

ex

(1) $V = V_1 \cup V_2$ a partition




(2) any edge it will connect only vertices between $V_1, V_2$

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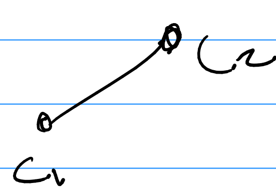
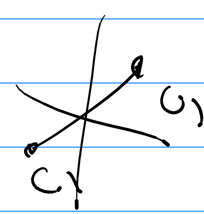
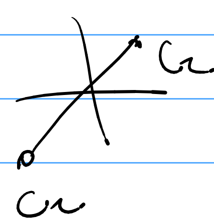
**Thm** Coloring thm — $G$ is bipartite iff

it is possible to assign one of 2 colors to every vertex and no two adj. vertices are same color

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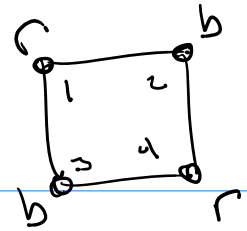
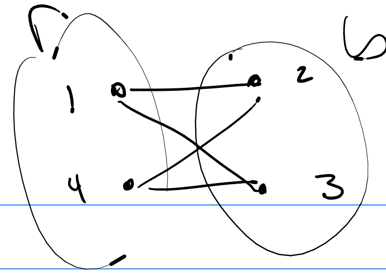
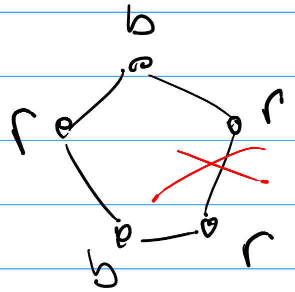
(2c)

$C_4$

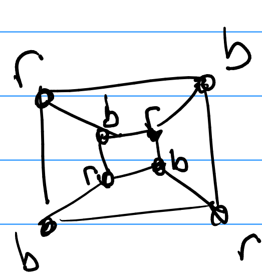


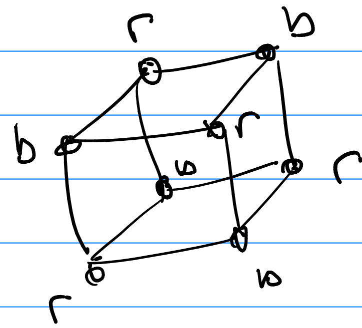


$C_5$



$Q_3$





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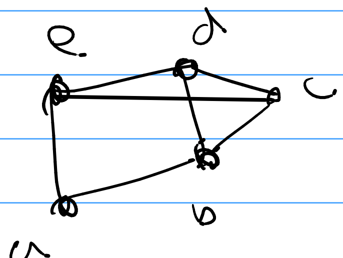
## Operations

① Subgraphs of $G = (V, E)$

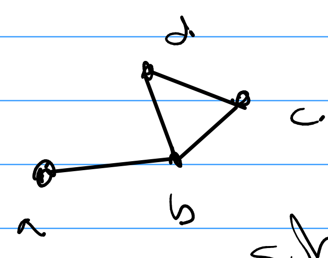
$$H = (W, F) \qquad W \subseteq V, \quad F \subseteq E$$

induced subgraph

$H = (W,$ take any edges of $E$ that have $W$ as both end points $)$




$G$




$H$ a subgraph

(2) <u>unions</u> $G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$

(3) remove edges, remove vertices (and incident edges)