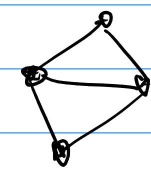
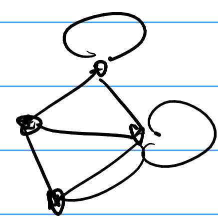
# Math 322

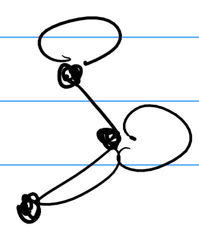
10.2 Ops on graphs $G=(V,E)$



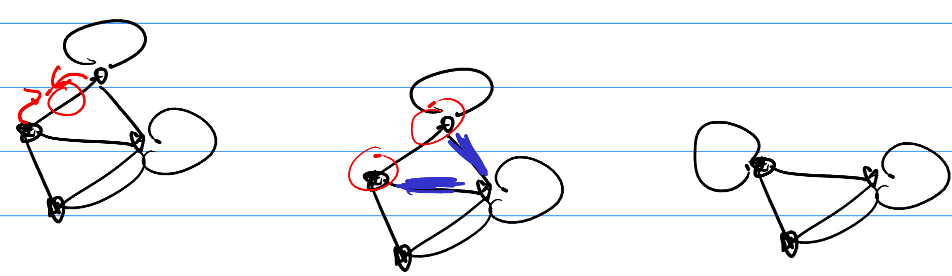
1. subgraphs or induced subgraphs
2. remove edges
3. add edges



4. remove vertices with incident edges



5. edge contraction

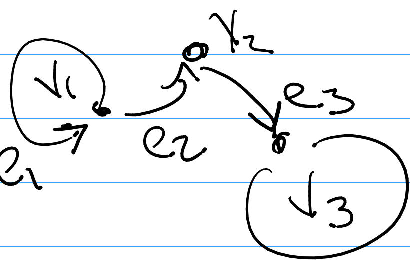


6. <u>Unions</u> $G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$

# 10.3 representing graphs and isomorphisms

## Represent Graphs

1. $G = (V, E)$

2. 


3. Adjacency Lists

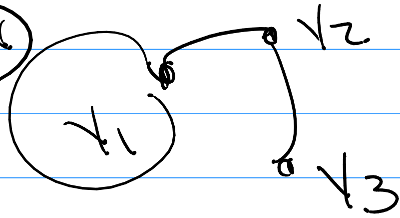
Undirected

| $V$ | all $v \in V$ adj. |
|---|---|
| $v_1$ | |
| $v_2$ | |
| $\vdots$ | |
| $v_n$ | |

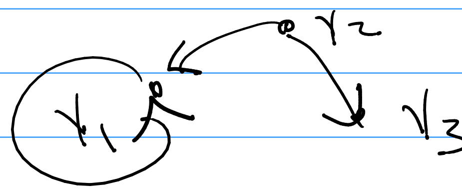
Directed

| $v$ (initial) | terminal |
|---|---|
| $v_1$ | |
| $v_2$ | |
| $\vdots$ | |
| $v_n$ | |

ex



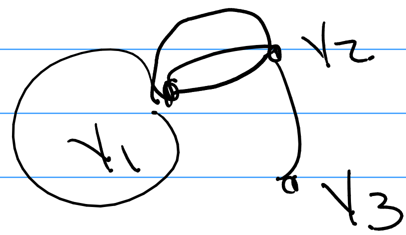

| | |
|---|---|
| $v_1$ | $v_1, v_2$ |
| $v_2$ | $v_1, v_3$ |
| $v_3$ | $v_2$ |

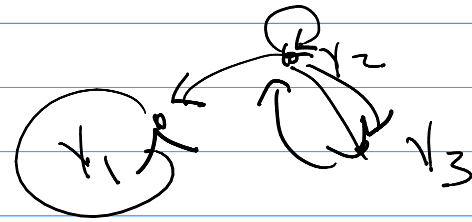



| | |
|---|---|
| $v_1$ | $v_1$ |
| $v_2$ | $v_1, v_3$ |
| $v_3$ | |

## ④ Adj. Matrix $M_A$ or $A_G = V\begin{bmatrix} \; \end{bmatrix}^{V}$

ex




$$A_G = \begin{array}{c|ccc} & v_1 & v_2 & v_3 \\ \hline v_1 & 1 & 3 & 0 \\ v_2 & 3 & 0 & 1 \\ v_3 & 0 & 1 & 0 \end{array}$$

$$A_G = \begin{array}{c|ccc} & v_1 & v_2 & v_3 \\ \hline v_1 & 1 & 0 & 0 \\ v_2 & 1 & 1 & 2 \\ v_3 & 0 & 1 & 0 \end{array}$$

→ Sparce vs Dense $|V| = n$

"few" edges "lots" of edges Matrix is $n^2$

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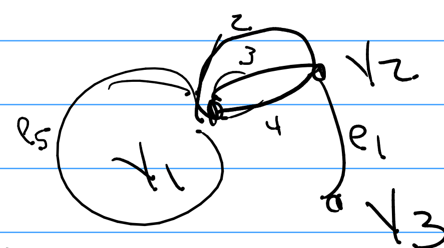
## ⑤ label vertices and edges




Incidence Matrix

$$I_G = \begin{array}{c|ccccc} & e_1 & e_2 & e_3 & e_4 & e_5 \\ \hline v_1 & 0 & 1 & 1 & 1 & 1 \\ v_2 & 1 & 1 & 1 & 1 & 0 \\ v_3 & 1 & 0 & 0 & 0 & 0 \end{array}$$

Note:

$(I_G \cdot I_G^T)$
v e  e v
3x5  5x3

$I_G^T \cdot I_G$
e v  v.e
3x3  3x5

"Same"

**Def** $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if $\exists f$ a bijection

Isomorphism → $f: V_1 \to V_2$ that preserves edges

$(a,b) \in E_1 \qquad (f(a), f(b)) \in E_2$

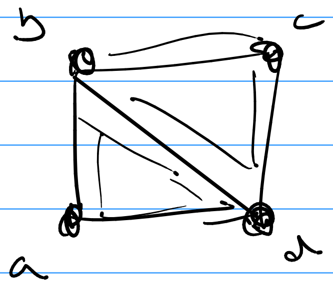
**Note** Invariants under Isomorphism

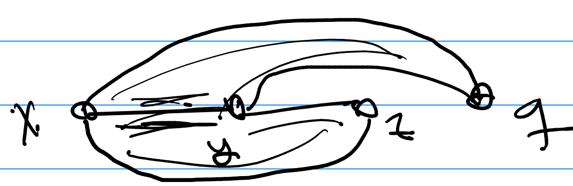
① $|E_1| = |E_2|$, $|V_1| = |V_2|$

② deg(v) must be preserved

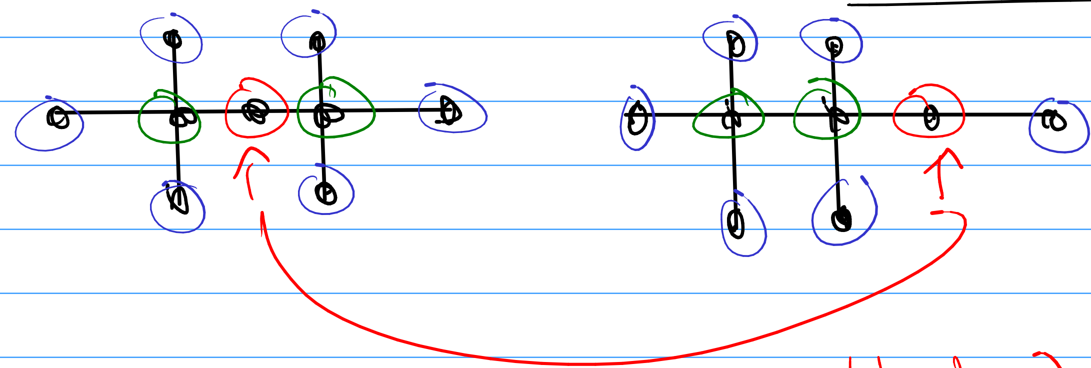
③ deg(v) in neighborhoods must be preserved

④ paths (seq of edges) must be preserved



① Show not-isomorphic → broken invariant

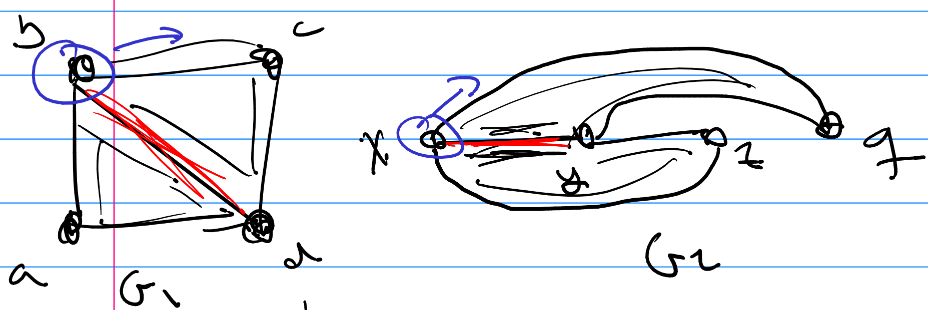


broken neighborhood

Not isomorphic

② Show isomorphic?

a) no broken invariants? (maybe)
b) find an isomorphism. (function)




$f$

$b \to x$
$c \to q$
$d \to y$
$a \to z$

c) Show Isomorphic

$$A_{G_1} = \begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} = A_{G_2} = \begin{array}{c|c} & \begin{matrix} z & x & q & y \\ f(a) & f(b) & f(c) & f(d) \end{matrix} \\ \begin{matrix} z\ f(a) \\ x\ f(b) \\ q\ f(c) \\ y\ f(d) \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$