

Math 322

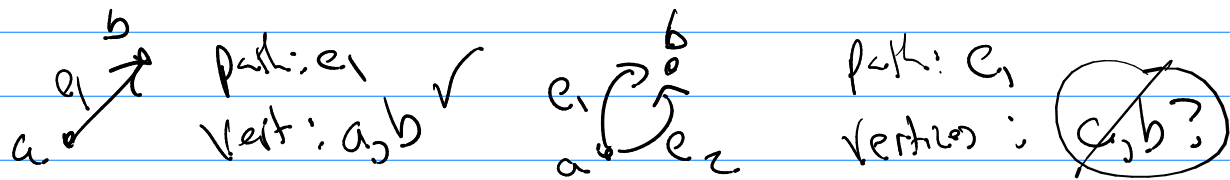
10.4 Connectedness in $G(V, E)$

Path: seq of n edges from v_0 to v_n

directed: $e_1 = (v_0, v_1), e_2 = (v_1, v_2), \dots, e_n = (v_{n-1}, v_n)$

undirected: $e_1 = \{v_0, v_1\}, e_2 = \{v_1, v_2\}, \dots, e_n = \{v_{n-1}, v_n\}$

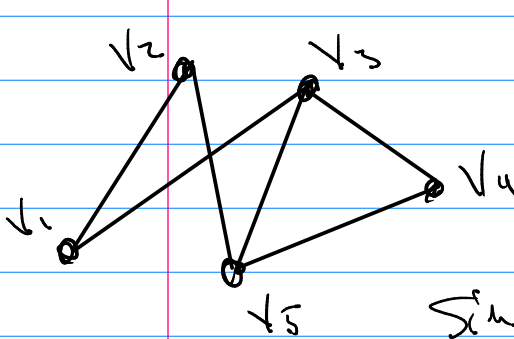
Note: if you don't confuse possible edges (multi-edge?)
 + list vertices $v_0, v_1, v_2, \dots, v_n$



terms

$v_0 = v_n$ and $n \geq 1 \rightarrow$ **Circuit**

if e_i in the seq. are all uniq \rightarrow **Simple**
 (don't reuse edges)



seq of edges as list of vertices notation

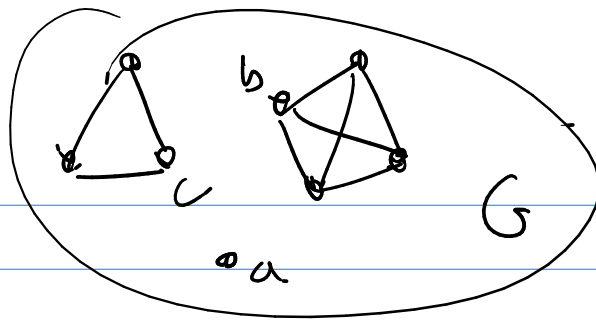
not a path: v_1, v_2, v_5, v_1, v_3
 (no edge between v_2 and v_5)

Simple circuit: $v_3, v_4, v_5, v_2, v_1, v_3$

non-simple circuit: v_2, v_5, v_2

not simple, not a circuit: v_1, v_3, v_1, v_2

Connected



Def (undirected) G is connected if there is a path between every pair of distinct vertices.

→ disconnected : not connected

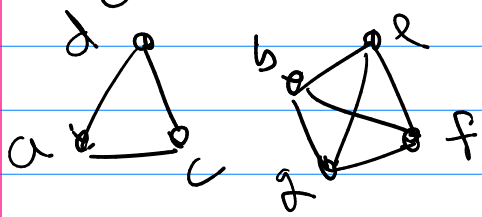
→ you can disconnect a graph if you remove vertices (edges and the subgraph is disconnected)

cut vertices, cut edges, vertex cut, edge cut
(one vertex) (one edge) (set of vertices) (set of edges)

Th^m there is a simple path between every pair of distinct vertices of a connected undirected graph.

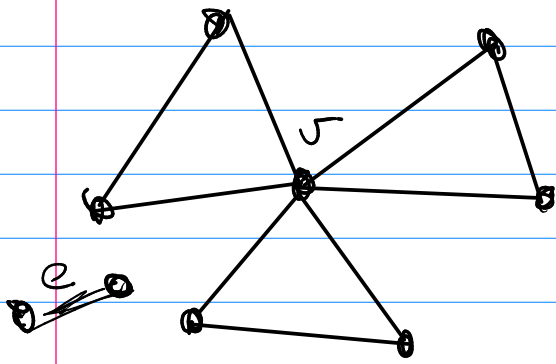
Connected Components

a disconnected graph is made of connected subgraphs (called connected components)



disconnected → 2 connected components
 $\{a, b, c\}$
 $\{b, c, d, f\}$

Connected graphs ... How strong is the connection?

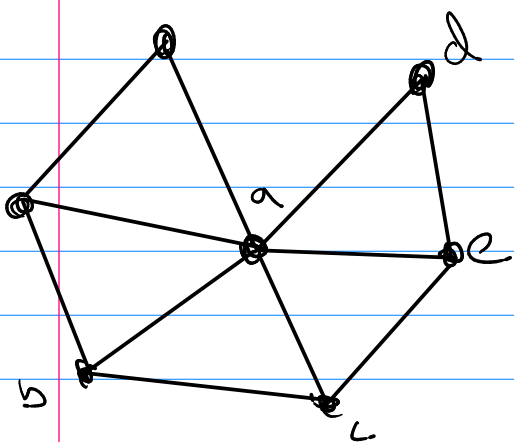


① Single "failure"

cut vertex: one vertex who removed will inc. # of connected comp.

cut edge: one edge who removed will inc # of connected comp.

ex) cut vertex is v
cut edge is e



need multiple to fail

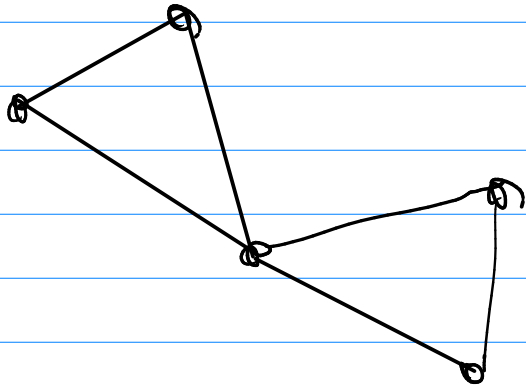
vertex cut: $\{a, b\}$

edge cut: $\{\{a, c\}, \{d, e\}\}$

Def: $\kappa(G)$ = minimal size for vertex cut
 $\pi(G)$ = minimal size for edge cut

ex) $\kappa(G) = 2$
 $\pi(G) = 2$

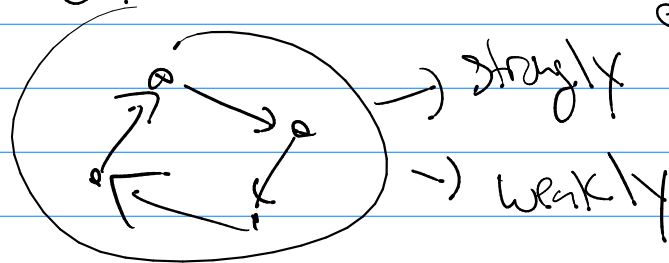
→ $\chi(G) \leq \tau(G) \leq \max_{v \in V} \deg(v)$



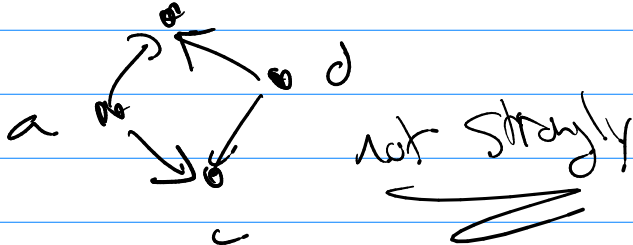
$\chi(G) = 1$
 $\tau(G) = 2$
 $\min \deg(v) = 2$

Directed Graphs

① Strongly Connected: path to and from each distinct pair



② IF connected when it's undirected



→ weakly connected