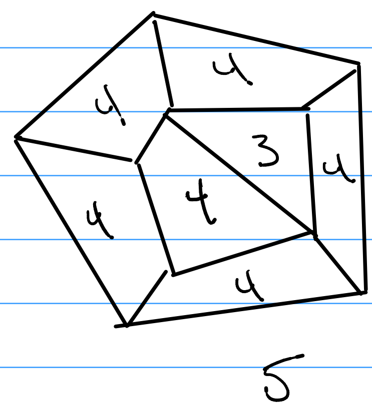
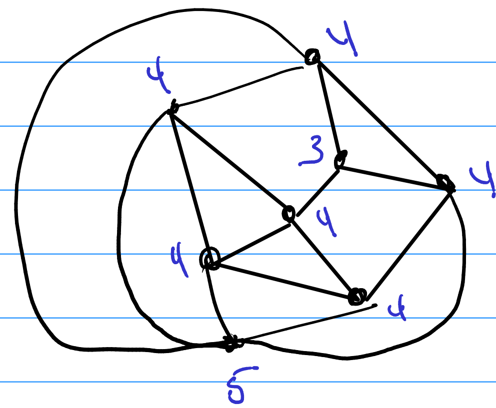
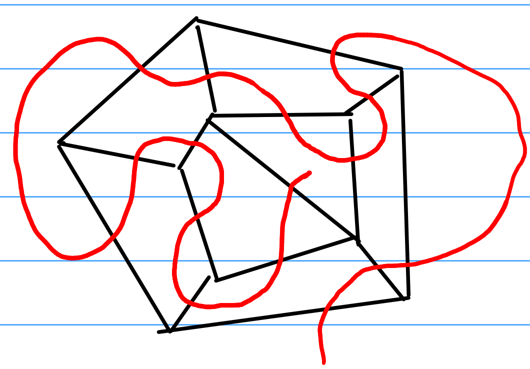
Math 322

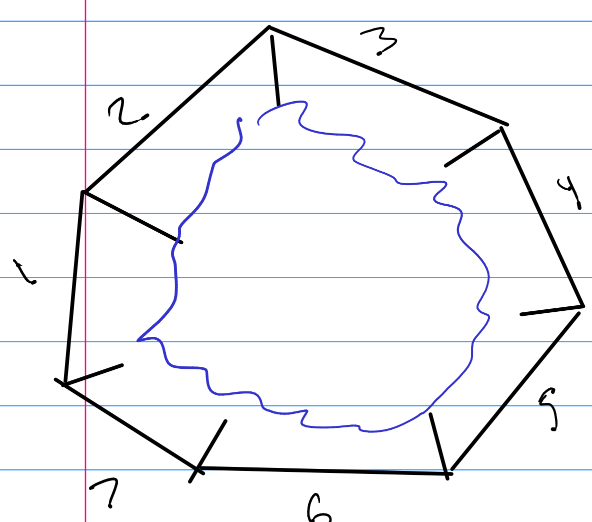
__Cut puzzles:__ Euler Circuit / Path





exactly two odd degree → Euler path
(not circuit)





handshake thm says the inside must have 1, 3, 5, or 2n+1 rooms with odd segments

Undirected

all even degree → Euler Circuit

only 2 odd degree → Euler Path

Directed

(1) $\forall v \ \deg^+(v) = \deg^-(v)$ → Euler Circuit

(2) all have $\deg^+(v) = \deg^-(v)$ → Euler Path

<u>except</u> (1) $\deg^+ = \deg^- + 1$ (plus (1) is the start and

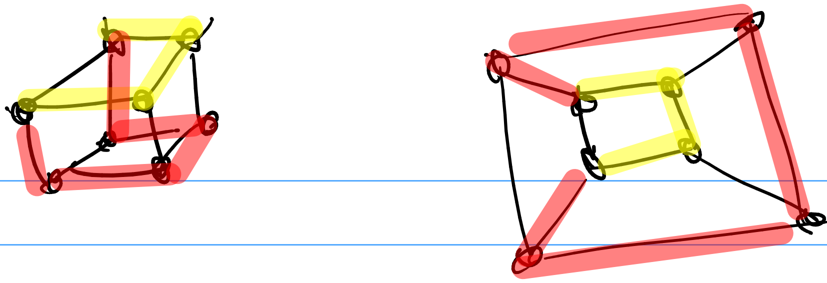
(2) $\deg^- = \deg^+ + 1$ (2) is the end)

Hamilton Path | Circuit

<u>Path</u> → simple path that visits each vertex once.

<u>Circuit</u> → simple path that visits each vertex once, except first vertex = last vertex.

Q_3



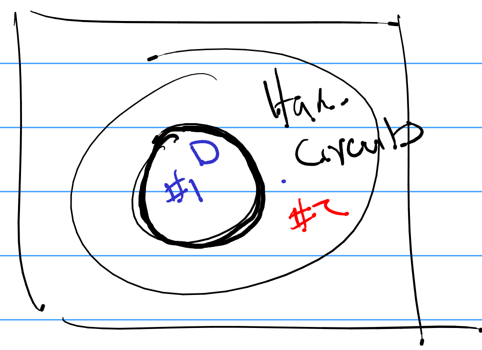
Thm □ iff △

Says □ ≡ △ (nec. and suff. conditions)

No simple nec. and suff. conditions exist for Hamilton circuits / paths.

We do have nice suff. conditions.

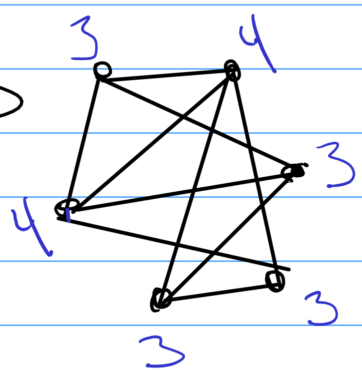
Thm □ $\Rightarrow$ △



$(|V| \geq 3)$

Dirac's $\deg(v) \geq |V|/2 \rightarrow$ Hamilton Circuit

ex

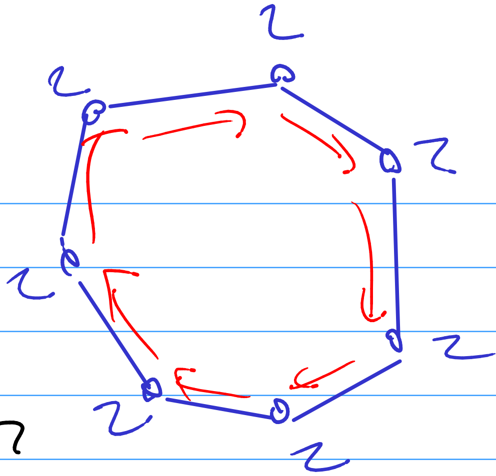


$|V| = 6$

$\deg(v) \geq 6/2 = 3$

$\Rightarrow$ Hamilton Circuit exists

ex



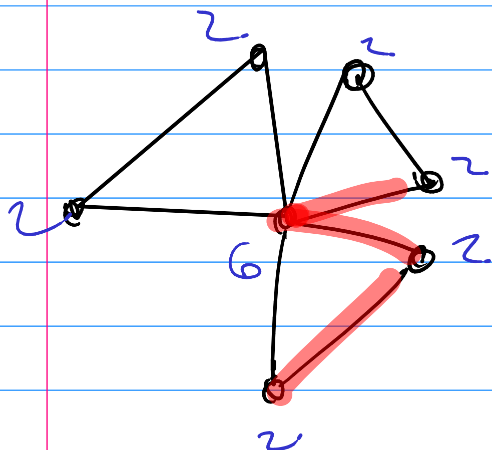
C_7

$|V| = 7$

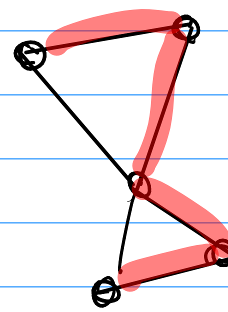
Dirac's $\deg(v) \geq 7/2 = 3.5$

$\Rightarrow$ $\boxed{\text{can't apply Dirac's}}$

and this one has a Hamilton Circuit.



$|V| = 7$

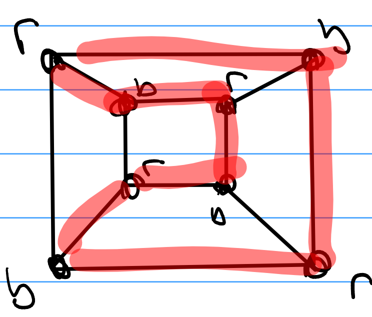


$\boxed{\text{ORE's thm}}$

$\deg(v) + \deg(u) \geq n$

u, v are non-adj.

$\rightarrow$ Hamilton Circuit.



$\deg(v) = 3$ $\quad |V| = 8$

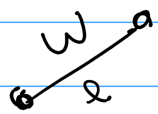
Dirac: $\deg(v) \geq 8/2 = 4$ X

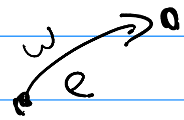
ORE's: $\deg(v) + \deg(u) \geq 8$

$3 + 3 \not\geq 8$ X

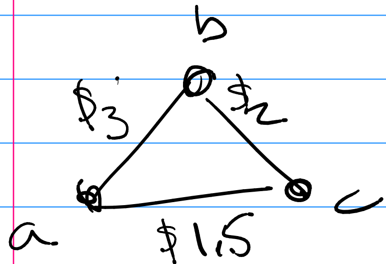
10.6 Shortest Paths.

$G=(V,E)$





weight function $f: E \to \mathbb{R}$



length of a path = sum of path's weights