Math 322

Qs/ 13.1 #11 example #7

$S \to C$, $C \to 0CAB$, $C \to \lambda$, $BA \to AB$

$0A \to 01$, $1A \to 11$, $1B \to 12$, $2B \to 22$

$\text{Set} = \{ 0^n 1^n 2^n \mid n = 0, 1, 2, \ldots \}$

$= \{ \lambda, 012, 001122, 000111222, \ldots \}$

$S \Rightarrow C \Rightarrow \lambda$

$S \Rightarrow C \Rightarrow 0CAB \Rightarrow 0AB \Rightarrow 01B \Rightarrow 012$

#11

$S \Rightarrow C \Rightarrow 0CAB \Rightarrow 00CABAB \Rightarrow 00ABAB$

$\Rightarrow 001BAB \Rightarrow 001ABB \Rightarrow 0011BB$

$\Rightarrow 00112B \Rightarrow 001122$

Show $\bar{0} = 1$ $\qquad 1 = x \vee \bar{x}$

$1 = 0 \vee \bar{0} = \bar{0}$

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a, b, c $\qquad$ example 4 p.761

a vs b

a c b c $\qquad$ a<b $\qquad$ a>b

a vs c $\qquad$ etc

a<c $\qquad$ a>c

b vs c $\qquad$ c, a, b

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parenthesize:

0 ops. (A) $C_0 = 1$

1 ops $(A+B)$ $C_1 = 1$

n ops: $(a_0) + (a_1 + a_2 + \dots + a_n)$

$\quad C_0 \cdot C_{n-1}$

or

$(a_0 + a_1) + (a_2 + \dots + a_n)$

$\quad C_1 \cdot C_{n-2}$

$$C_n = C_0 \cdot C_{n-1} + C_1 \cdot C_{n-2} + \dots + C_{n-1} \cdot C_0$$

$1, 1$

$C_2 = 1 \cdot 1 + 1 \cdot 1 = 2$

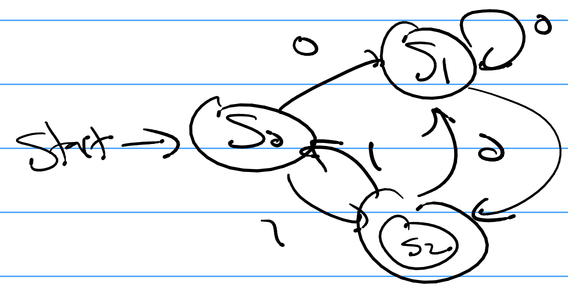
$C_3 = 1 \cdot 2 + 1 \cdot 1 + 2 \cdot 1 = 5$

$\vdots$

13.3 F.S.A. (Finite State Automaton)

$M = (S, I, f, s_0, F)$

ex.



f	0	1
S_0	S_1	S_2
S_1	S_1	S_2
S_2	S_1	S_0

$f: S \times I \to S$ transition function

f: $(S_0, 0) \to S_1$ $\quad (S_1, 0) \to S_1$ $\quad (S_2, 0) \to S_1$

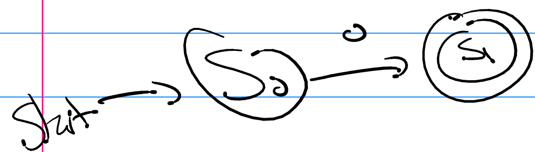
$(S_0, 1) \to S_2$ $\quad (S_1, 1) \to S_2$ $\quad (S_2, 1) \to S_0$

$(S_0, 0) \to S_1$
$(S_0, 1) \to S_2$
$(S_1, 0) \to S_1$
$(S_1, 1) \to S_2$
$(S_2, 0) \to S_1$
$(S_2, 1) \to S_0$

$f: S \times I \to S$

M is a Deterministic F.S.A.

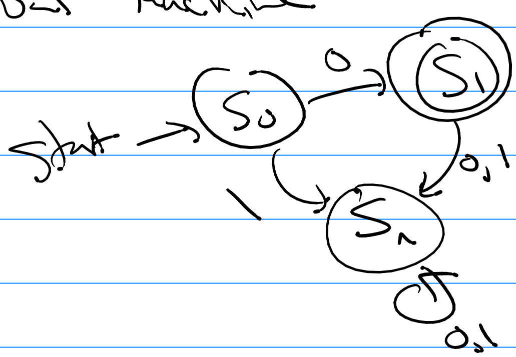
Short-hand / Lazy versions of F.S.A (state diagram)



Know $I = \{0, 1\}$

	f: 0	1
S_0	S_1	
S_1		

the "actual" machine



	f: 0	1
S_0	S_1	S_n
S_1	S_n	S_n
S_n	S_n	S_n

Language Recognition

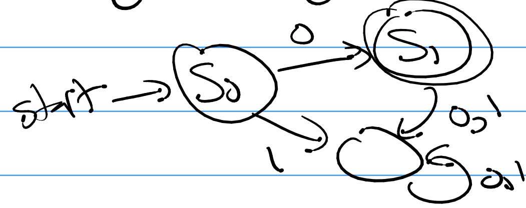
M recognizes x an input string if x takes S_0 to a final state

$L(M)$ = set of all recog. strings

ex

$L(M) = 0$

vs



$L(M) = 0$

if $L(M_1) = L(M_2)$ we say M_1, M_2 are equivalent.

$L(M) =$ set all all strings that take S_0 to a final state

Notation for strings

$A = \{x_1, x_2, .. x_n\}$

A^n = set of all strings that concatenate n elements of A.

ex $\{01, 001\}^3 = \{010101, 001001001$
$0101001, 0100101, 0010101$
etc $\}$

$A^0 = \{\lambda\}$

$A^* = A^0 \cup A^1 \cup A^2 \cup \ldots$ (Kleene closure)

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ex all bit-strings $\boxed{\{0,1\}^*}$

ex all strings with a 11 $\boxed{\{0,1\}^* 11 \{0,1\}^*}$

ex $11^* = 1\{\lambda, 1, 11, 111, ..\} = \boxed{1^n, n = 1, 2, 3, ..}$

ex same amount of 1's before the exact same amount of 0's
$1^n 0^n, n = 0, 1, 2, ..$

$L(M) = ?$

Final States

s_0	λ
S_2	$10^*,\ 00^*10^*$