# Math 322

## Turing Machines

$T = (S, I, f, s_0)$

$f$: partial function as a set of 5-tuples

Ex) ① remove the 1st two ones (by making them zeros) and then halt in a final state

$$01001001 \xrightarrow{T} 00000001$$

start tape — end tape (T recognizes it)

$S = \{s_0, s_1, s_N, s_{11}\}$

$f = \{ (s_0, 0, s_0, 0, R), (s_1, 0, s_1, 0, R), (s_N, 0, s_N, B, R),$
$(s_0, 1, s_1, 0, R), (s_1, 1, s_{11}, 0, R),$
$(s_0, B, s_N, B, R), (s_1, B, s_N, B, R)$

$s_{11}$

0 0 0 0 0 0

$s_0$

0 1 0 1 0 0

$s_0$

0 1 0 1 0 0

$s_1$

0 0 0 1 0 0

$s_1$

0 0 0 1 0 0

$s_{11}$

0 0 0 0 0 0 Halts final state

$s_0$

-- B 0 0 0 B --

$s_0$

-- B 0 0 0 B --

$s_0$

-- B 0 0 0 B --

$s_0$

-- B 0 0 0 B B --

$s_N$

-- B 0 0 0 B B -- Halts in a non-final state

## Number theoretic Functions:

$$f(n_1, n_2, \ldots, n_k) = n$$

a) unary $0 = 1$, $1 = 11$, $2 = 111$, ...

b) seperator $*$

[ex] $2 + 3 = 5$

start tape $\xrightarrow{T}$ end tape (halts in final state)

$\underbrace{111 * 1111}_{8 \text{ symbols}} \longrightarrow \underbrace{111111}_{6 \text{ symbols}}$

$1 + 2 = 3$

$\underbrace{11 * 111}_{6 \text{ symbols}} \xrightarrow{T} \underbrace{1111}_{4 \text{ symbols}}$

$$f = \{ (s_0, 1, s_1, B, R), (s_{11}, 1, s_{11}, 1, R), (s_1, 1, s_{11}, B, R), (s_{11}, *, s_F, 1, R), (s_1, *, s_F, B, R), \}$$

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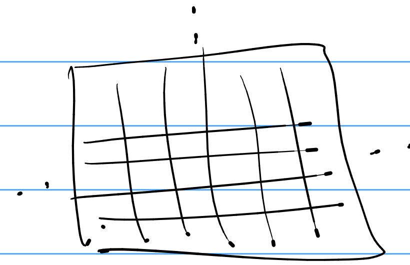
## Church-turing thesis

any problem that can be solved with an effective algorithm $\rightarrow$ there exists a turing machine that solves it.

[ex] is $n$ prime?

More powerful turing machines?

a) add memory / heads



c) remove directions?

d) <u>non-det.</u> $(S_0, 1, S_0, 1, R)$
$(S_0, 1, S_1, 0, R)$

→ $\boxed{\text{No power changes}}$

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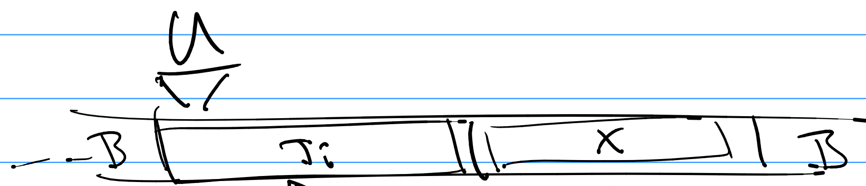
Any special turing machines?

① Universal Turing Machine : U

$T_i(X)$ = output on tape




encoding of any T

input I would give $T_i$

$$U(T_i, X) = T_i(X)$$