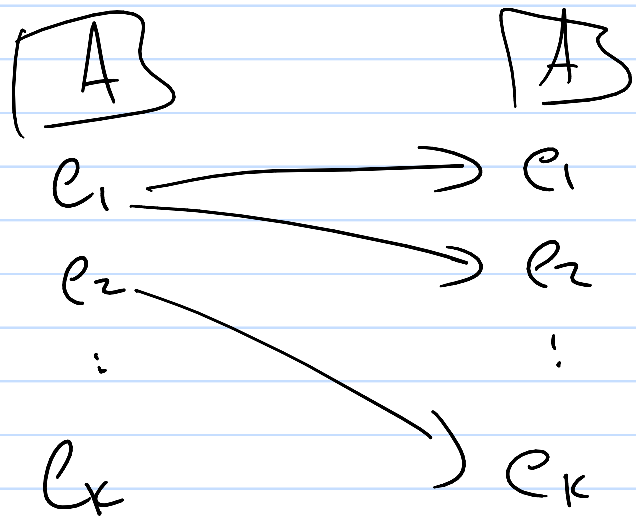
Math 322

Relations on set A (subset of $A \times A$)

set A is domain and codomain

Arrow diagram



digraph
$\shortparallel$
directed graph

$S_1 = \{a, e, i, o, u\}$

$S_2 = \{a, b, c, d, e\}$

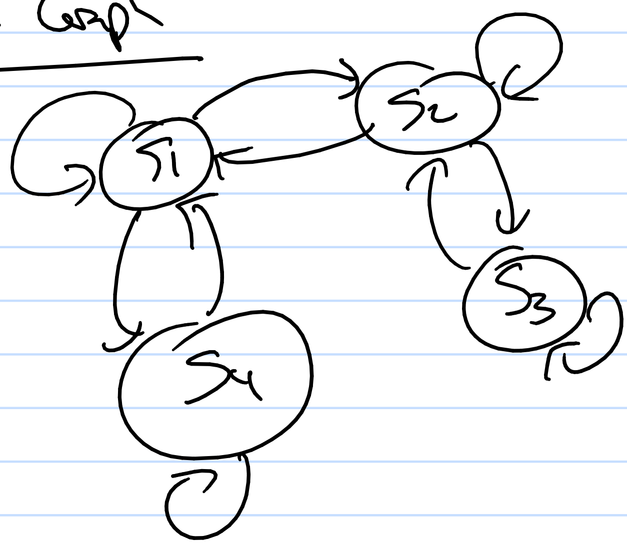
$S_3 = \{b, c, d, f, g\}$

$S_4 = \{m, n, o, p, \ldots, z\}$

Intersection Graph

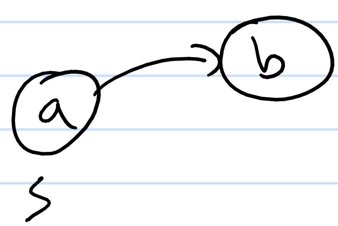
Adj. list

S_1	S_1, S_2, S_4
S_2	S_1, S_2, S_3
S_3	S_3, S_2
S_4	S_1, S_4



$\{(S_1, S_4)$
(S_1, S_1)
(S_1, S_2)
(S_2, S_1)
(S_2, S_2)
(S_2, S_3)
(S_3, S_2)
(S_3, S_3)
(S_4, S_1)
$(S_4, S_4)\} = r$

6.1 Adjacency



$a r b$

$(a,b) \in r$

a is adjacent to b

b is adjacent from a

Adjacency list

set A	list all vertices adj. from e
e_1	
e_2	
$\vdots$	
e_k	

Adj. Matrix

$$M_r = [R_{ij}]$$

$$R_{ij} = \begin{cases} 0 & \text{if } a_i \not r\, b_j \\ 1 & \text{if } a_i\, r\, b_j \end{cases}$$

bits

bit operations

$0 + 0 = 0$ $\qquad$ $0 \cdot 0 = 0$

$1 + 0 = 1$ $\qquad$ $1 \cdot 0 = 0$

$0 + 1 = 1$ $\qquad$ $0 \cdot 1 = 0$

$1 + 1 = 1$ $\qquad$ $1 \cdot 1 = 1$

ex

Adj. list

S_1	S_1, S_2, S_4
S_2	S_1, S_2, S_3
S_3	S_3, S_2
S_4	S_1, S_4

$$M_r = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$R_1 \cup R_2$ $\qquad$ $M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2}$

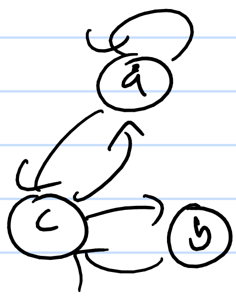
$R_1 \cap R_2$ $\qquad$ $M_{R_1 \wedge R_2} = M_{R_1} \wedge M_{R_2}$

$\underbrace{r_1 r_2}$ $\qquad$ $M_{r_1 r_2} = M_{r_1} \cdot M_{r_2}$

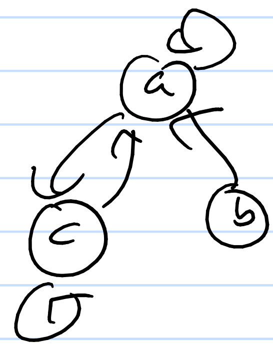
composition

$$M_{r_1 r_2} = \begin{array}{c} \\ a \\ \\ \\ \end{array}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{c} a \\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} 3 & & \\ & & \\ & & \end{bmatrix}$$

M_{r_1}



M_{r_2}



$$1\cdot 1 + 0\cdot 1 + 1\cdot 1 = 1 + 0\cdot 1 + 1 = 1$$

$$= 2$$

Composition:

$$r_1 r_2 r_3 \rightsquigarrow M_{R_1} M_{R_2} M_{R_3}$$

$$r_1 r_1 r_1 \rightsquigarrow M_{R_1} M_{R_1} M_{R_1}$$

$$r_1 r_1 r_1 = r_1^3 \qquad M_{R_1} M_{R_1} M_{R_1} = (M_{R_1})^3$$

So r_1^n is $(M_{R_1})^n$

given a digraph

Def: a path is a seq of edges from one vertex to any other vertex

a) if start of path = end of path

→ call the path a circuit

b) if no edge is used more than once → call it simple

c) length = # of edges

$$(M_r)^n = [\;\bigcirc\;]$$

path of length n from a_i to b_j if $\neq 0$