Math 322 Prove $p \to q$

Q's 6.1 #6

set of ordered pairs

$a r_1 b$ means $(a,b) \in r_1$

r_1, r_2, r_3 are relations a set A

composition

Prove: if $r_1 \subseteq r_2 \to \boxed{r_1 r_3} \subseteq \boxed{r_2 r_3}$

subset

if in set 1, you are in set 2

set 1 $\subseteq$ set 2

Note: "Understanding"

① read

② plan

③ do plan

Scratch paper

if $a r_1 b$ then $a r_2 b$

if $c r_1 r_3 d$ then $c r_2 r_3 d$

$c r_1 M \wedge M r_3 d \to c r_2 M \wedge M r_3 d$

b/c

$c r_1 M$ and $M r_3 d$ $\boxed{\text{so}}$ $c r_2 M$ and $M r_3 d$

Statement: if $r_1 \subseteq r_2$, then $r_1 r_3 \subseteq r_2 r_3$

Pf: assume $r_1 \subseteq r_2$ it says $(a,b) \in r_1$ then $(a,b) \in r_2$

show $r_1 r_3 \subseteq r_2 r_3$

for any a, b

now,

there exists an M

if $(c,d) \in r_1 r_3$ says $c r_1 M$ and $M r_3 d$

by $(c,d) \in r_1 r_3$

show $(c,d) \in r_2 r_3$

and from above $c r_1 M \to c r_2 M$

therefore: $c r_1 M$ and $M r_3 d \to c r_2 M$ and $M r_3 d$

We have shown $(c,d) \in r_1 r_3 \rightarrow (c,d) \in r_2 r_3$

therefore $r_1 r_3 \subseteq r_2 r_3$

$\blacksquare$ Q.E.D.

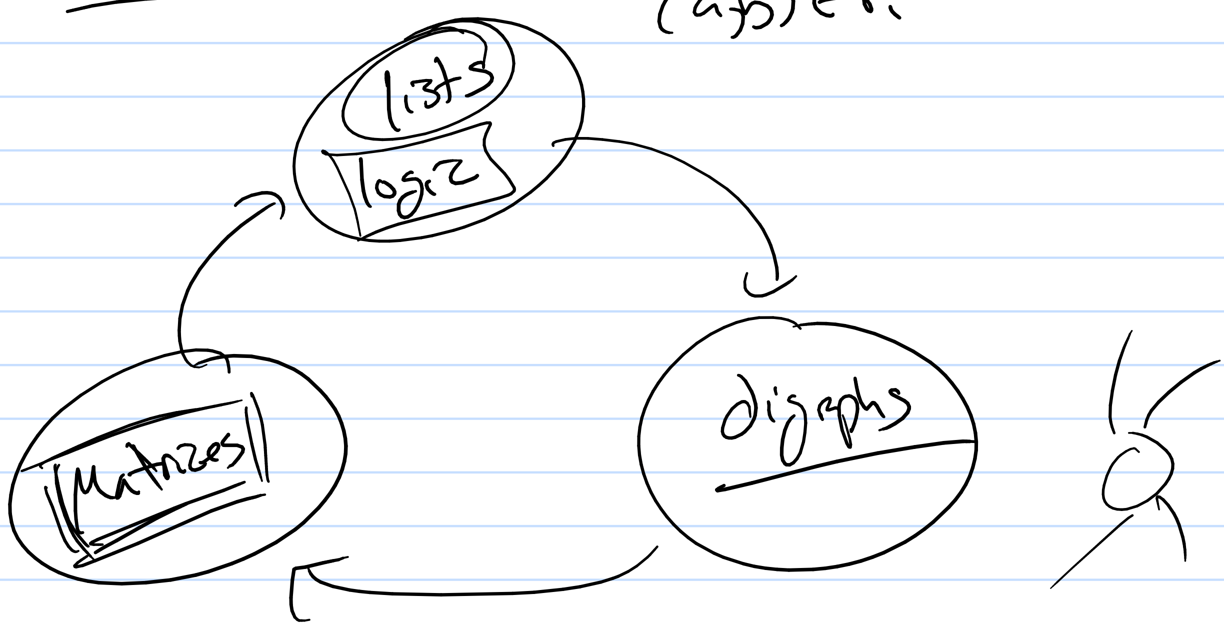
6.1 relations

6.2
~~6.3~~
6.4

representing relations

$(a,b) \in r_1$

relation



Applications of Relations 6.3

① Equivalence Relations

② Partial Orderings (Posets)

→ total orderings

→ well orderings

Properties of Relations   relation r on set A.

① reflexive property $\forall e \in A \, (e \, r \, e)$

list: $(e_1 \, r \, e_1)$
$(e_2 \, r \, e_2)$
$\vdots$

digraph 

all vertices have a loop to itself.

Matrix

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$$

all ones a diagonal

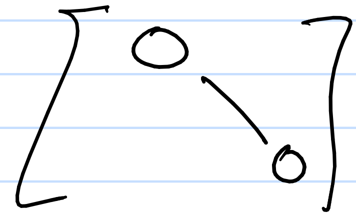
② irreflexive $\forall e\,(e \not r e) \equiv \neg \exists e\,(e r e)$

list: never have an $e_1 r e_1$

digraph

no loops anywhere.

Matrix



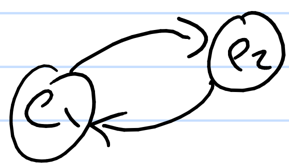
0's on diagonal

③ Symmetric $\forall e_1 \forall e_2\,(e_1 r e_2 \rightarrow e_2 r e_1)$

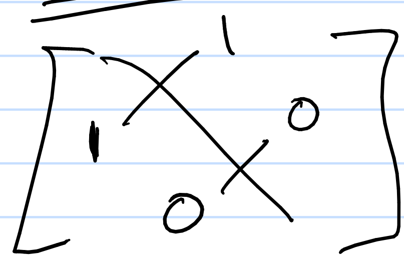
list:

(a,b) have (b,a)

digraph



Matrix



$M^T = M$

④ Anti Symmetric $\forall e_1 \forall e_2\,(e_1 r e_2 \wedge e_2 r e_1 \rightarrow e_1 = e_2)$

$\equiv \forall e_1 \forall e_2\,(e_1 \neq e_2 \rightarrow \boxed{e_1 \not r e_2 \vee e_2 \not r e_1})$

$= \forall e_1 \forall e_2\,(e_1 \neq e_2 \rightarrow \neg(e_1 r e_2 \wedge e_2 r e_1))$

list
a≠b (a,b) never have (b,a)

digraph

Matrix