Math 322

6.4 (1, 3, 4, 7, 11)   Due Sept 14
6.5 (1, 3, 5)

(ex) Relations on set $\{a, b, c\}$

$R_1 = \{ (a,a), (a,b), (b,c), (c,a) \}$

$R_2 = \{ (a,c), (b,a), (b,b), (c,b) \}$

$$M_{r_1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad M_{r_2} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

R_1R_2 as matrix

$$M_{r_1r_2} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

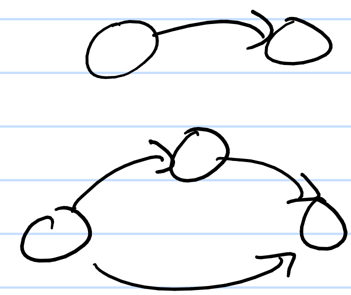
Visualize a partial ordering: Hasse diagram

<u>Modification of a digraph.</u>

<u>digraph</u>

partial ordering needs (1) reflexive — loops

(2) antisym



(3) transitive

<u>Hasse diagram</u>

#1 we <u>know</u> it is reflexive.

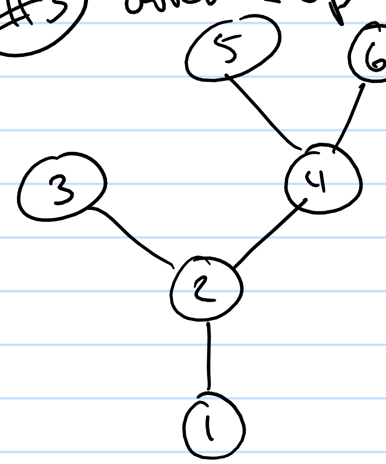
So don't draw loops (assume they are there)

#2 We <u>know</u> it is transitive

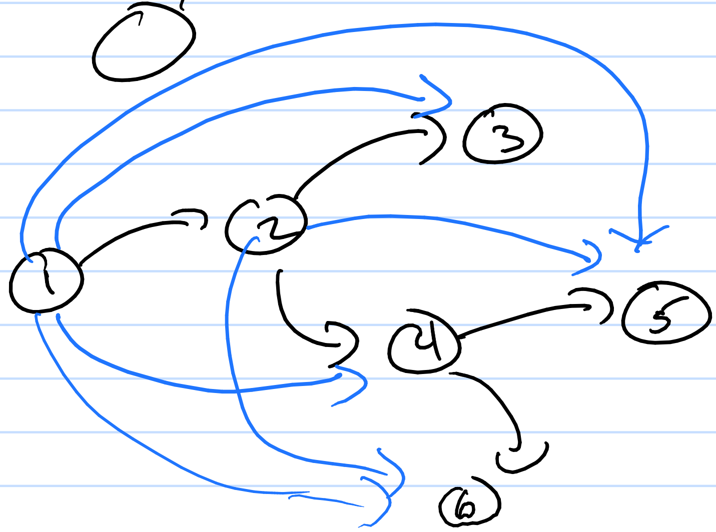
So don't draw transitive edges

What do we "know"?

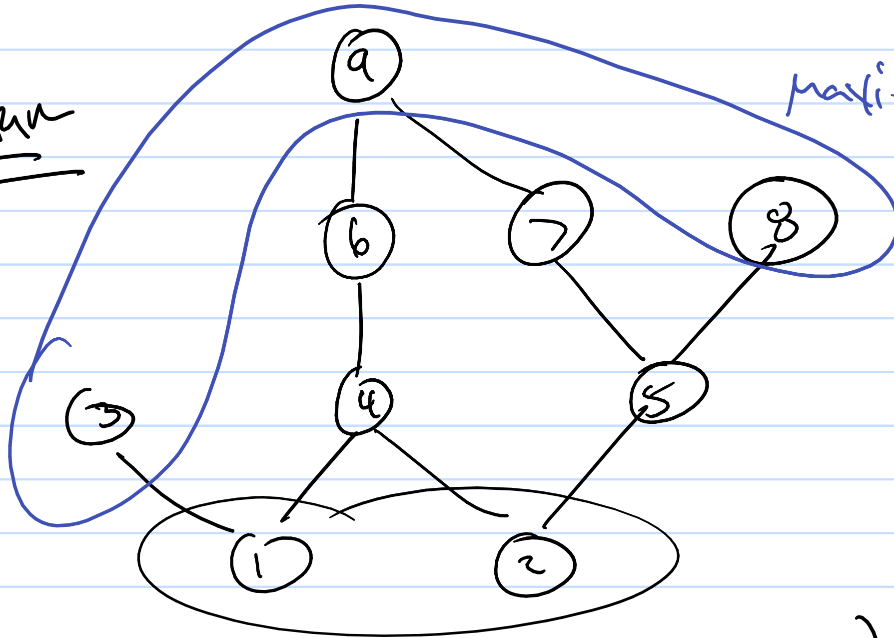
#3 arrow up



ex



Hasse diagram



maximal: no edge above

minimal : no edge below

least = the single unique minimal (if only one)

greatest = unique (only one) maximal

consider a subset of elements

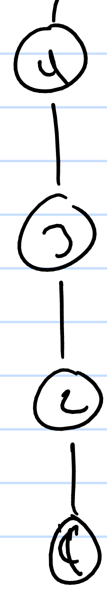
lower bound = set of all elements with a path to the subset elements

$\boxed{\text{Find}}$ greatest lower bound?

upper bound = set of all elements with a path from the subset elements

$\boxed{\text{Find}}$ least upper bound?

Well order



6.5 If a relation is not reflexive? / symmetric? / transitive? can we make a New relation by adding edges that has the property we are interested in?

Given r and it is not reflexive

so take $r \cup \{$ stuff to make it reflexive $\}$ ← as small as possible

$= $ New relation that is reflexive

↑ reflexive closure of r

ref. closure $= \boxed{M_r \vee I}$ → $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

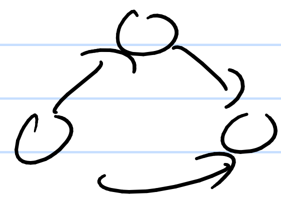
Symmetric closure

if $a \, r \, b$ we must have $b \, r \, a$

Symmetric closure = $\boxed{M_r \vee M_r^T}$

Transitive Closure ?

Th^m: r on set A is transitive if and only if $\forall n \; r^n \subseteq r$



$$r^+ = \underbrace{r \cup r^2 \cup r^3 \cup r^4 \cup \dots}$$

connectivity relation

$$\boxed{M_{r^+} = M_r \vee M_r^2 \vee \dots \vee M_r^{n}}$$

number of elements