Math 322

Q's 6.3 #11 6.5 #5

6.3 #1 is $R = \{(p,q) \mid p,q$ are propositions and $p \to q\}$

Poset? — reflexive? — antisymmetric? — transitive?

reflexive- $\forall e\,(eRe) \equiv$ for all propositions, e, $(e \to e)$ a tautology?

$$(e \to e) \equiv \neg e \vee e \equiv T$$

Yes!

or

e	$e \to e$
T	T
F	T

antisymmetric $\forall e_1 \forall e_2\,(e_1Re_2 \wedge e_2Re_1 \to e_1 = e_2)$ "same"

$$\left(\left[(e_1 \to e_2) \wedge (e_2 \to e_1)\right] \to (e_1 \equiv e_2)\right) \overset{?}{\equiv} T$$

$(e_1 \Leftrightarrow e_2) \equiv T$

$$(\boxed{e_1 \Leftrightarrow e_2}) \Rightarrow (e_1 \Leftrightarrow e_2)$$

T

Yes

transitive $(e_1 R e_2 \wedge e_2 R e_3) \to (e_1 R e_3) \overset{?}{\equiv} T$

$$\left[(e_1 \to e_2) \wedge (e_2 \to e_3)\right] \to (e_1 \to e_3) \overset{?}{=} T$$

direct proof: $(e_1 \to e_2) \wedge (e_2 \to e_3)$ is true

$(\neg e_1 \vee e_2) \wedge (\neg e_2 \vee e_3)$ is true

$\Big\downarrow$ todo

$\neg e_1 \vee e_3$

$(e_1 \to e_3)$ is true

yes

---

q_i: "$n < i$ on $\mathbb{N}$"

q_1 : "$n < 1$ on $\mathbb{N}$"

q_2 : "$n < 2$ on $\mathbb{N}$"

q_3 : "$n < 3$ on $\mathbb{N}$"

$q_1 \to q_2$?

"$(n<1) \to (n<2)$" true

so (q_1, q_2)

Pairs: (q_1, q_1), (q_1, q_2), (q_1, q_3), ...

(q_2, q_2), (q_2, q_3), ...

|

q_3

|

q_2

|

q_1

6.5 #5

Zero-one matrices

closures of S a relation.

reflexive closure $\boxed{M_S + I}$ Make diagonal all ones

sym. closure $\boxed{M_S + M_S^T}$ — transpose

$\boxed{M_S + M_S^{-1}}$

transitive closure

from thm R is transitive iff

$\forall n \quad R^n \subseteq R$

Connectivity relation for relation S $\quad S^+ = S + S^2 + S^3 + S^4 + \cdots$

if S is a set $|A| = \boxed{n}$ (finite)

know

S^k holds all e_i to e_j connected by a path of length K.

$S^+ = S + S^2 + S^3 + \cdots + S^n$ $\boxed{\text{+ have circuits}}$

Finding transitive closure : S^+

$\boxed{\text{technique \#1}}$ $S^+ = S + S^2 + S^3 + \cdots + S^n$

$S^2 \uparrow S \circ S$, $S^3 \uparrow S \circ S^2$, $\uparrow S \circ S^3$, $S^n \uparrow S \circ S^{n-1}$

$\boxed{\text{technique \#2}}$

consider the seq.

$S_1 = S$

$S_2 = S + S^2 = S(I+S) = S_1(I+S_1)$

$S_3 = S + S^2 + S^3$

$S_4 = \underbrace{S + S^2}_{S_2} + \underbrace{S^3 + S^4}_{??}$

Notice $S_2 \cdot S_2$

$(S+S^2)(S+S^2)$

$S^2 + S^3 + S^3 + S^4$

$S^2 + S^3 + S^4$

$S_4 = \underbrace{S + S^2}_{} + \underbrace{S^2 + S^3 + S^4}_{}$

$S_4 = S_2 + S_2 \cdot S_2$

$= S_2 (I + S_2)$

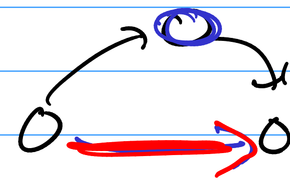
So
$$\left[\begin{array}{l} S_1 = S \\ S_2 = S_1(I+S_1) \\ S_4 = S_2 \circ (I+S_2) \\ S_8 = S_4 \circ (I+S_4) \\ S_{16} = S_8 \circ (I+S_8) \\ \\ S_{2^{k+1}} = S_{2^k}(I + S_{2^k}) \end{array}\right.$$

loop

when $2^{k+1} > n$ $\quad S_{2^{k+1}} = S^+$

Technique #3

Warshall's algorithm ?



$$M_S = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$n\ |A| = 4$

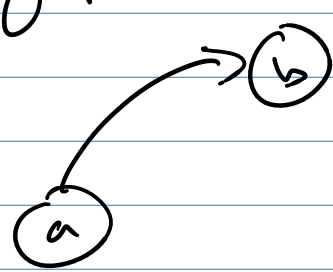
$$W_1 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \qquad W_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$W_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \qquad \boxed{W_4 = S^+ = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}}$$

Graphs! chapter #9

digraphs for relationships

(a,b)



graph theory : study of

$G = (V, E)$

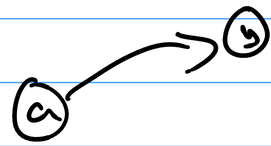
V : non-empty set of vertices

E : set of edges

$$G = (V, E)$$

edge is ordered pairs of vertices

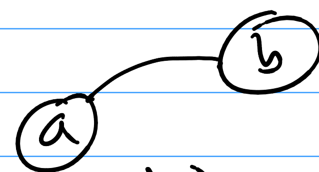
(a, b)



called **directed graphs**

edge is unordered pair of vertices

$\{a, b\}$



undirected graphs

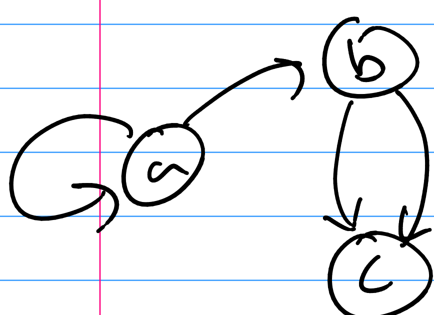
terms

graph types

1. simple directed graph
   - loops = ok
   - no multiple edges
2. directed multigraph
   - loops = ok
   - multi = ok

graph types

1. simple undirected graph
   - has no loops
   - no multiple edge
2. undirected multigraph
   - no loops
   - multiple edges = ok
3. pseudograph
   - loops = ok
   - multi = ok



directed multigraph

count edges $|E| = ?$

count vertices $|V| = ?$

directed graph

In Degree = # of edges into the vertex

= $\text{indeg}(v)$

= $\deg^-(v)$

Out degree = # of edges from the vertex

= $\text{outdeg}(v)$

= $\deg^+(v)$

$$|E| = \sum \deg^-(v) = \sum \deg^+(v)$$

undirected graph

Degree = # of edges connected to v

= $\deg(v)$

$$\boxed{\sum_{v \in V} \deg(v) = 2|E|}$$

also says we have even #'s of odd degrees