Math 322

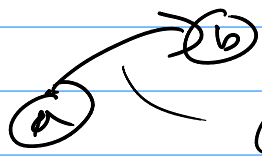
9.1 (5, 6, 8)
9.2 (1)
9.3 (3)
9.4 (2, 1, 5, 6d, 10)

Graphs

$G = (V, E)$

directed

directed edges



(a,b)

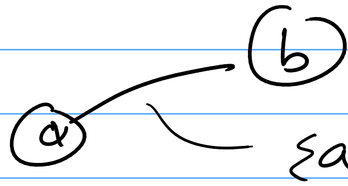
types
- Simple
- multi

in degree $\deg^-(v)$
out degree $\deg^+(v)$

thm $\sum_{v \in V} \deg^+(v) = \sum_{v \in V} \deg^-(v) = |E|$

undirected

undirected edges

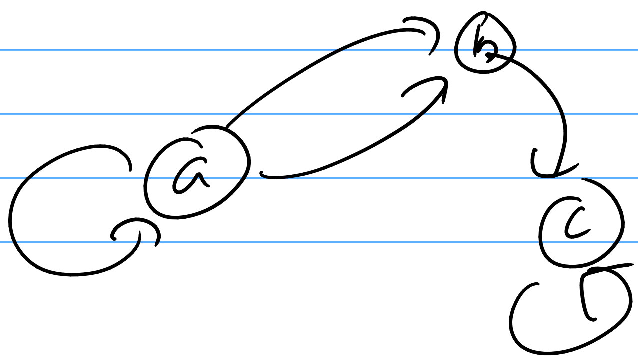


{a,b}

types
- simple
- multi
- psuedo

$\text{degree}(v) = \deg(v)$

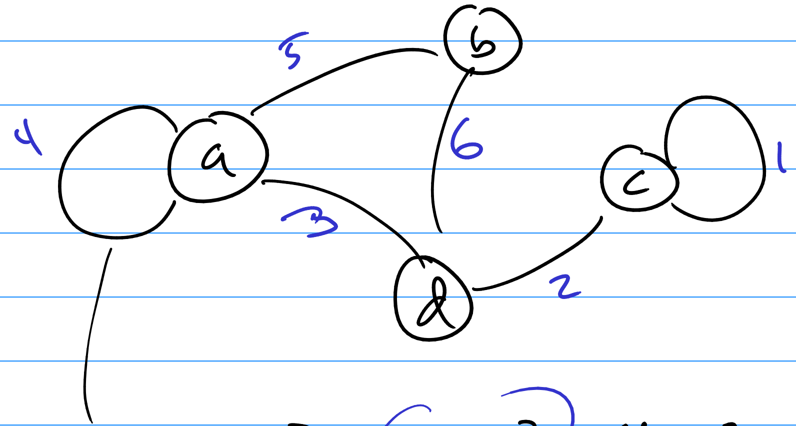
thm $\sum_{v \in V} \deg(v) = 2|E|$



Path: Sequence of edges

- length of a path = number of edges in seq.
- Simple path: in the seq, no edge is used more than once
- Circuit: seq of edges that start and end at the same vertex.

(ex)



deg(a) = 4
deg(b) = 2
deg(c) = 3
deg(d) = 3

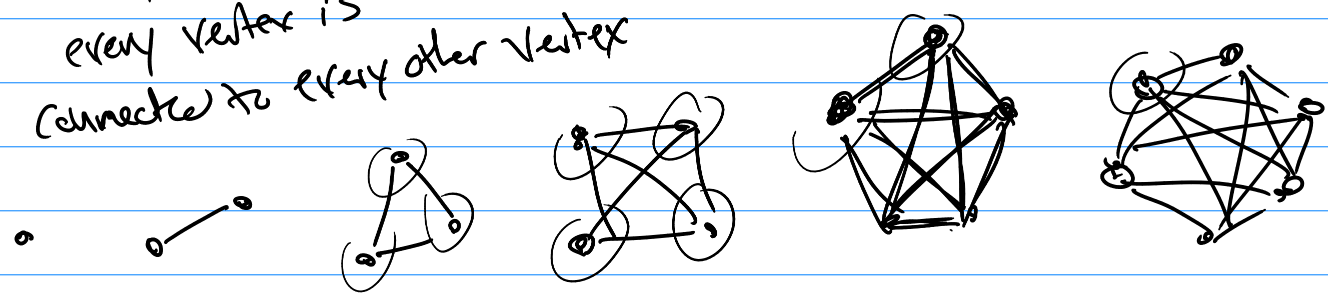
(ex) path {a,a}, {a,a}, {a, b}, {b,d}

list vertices: a, a, a, b, d

Special Simple Undirected Graphs

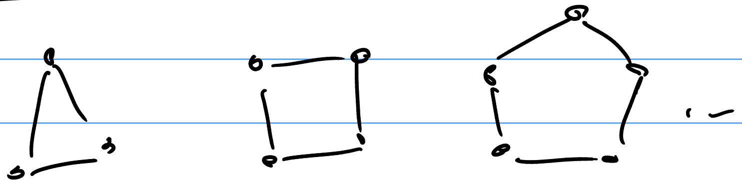
(1) Complete Graph, K_n n = number of vertices

every vertex is connected to every other vertex



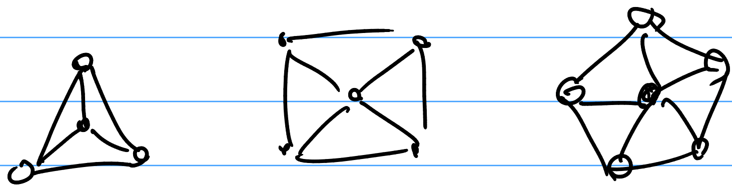
	K_1	K_2	K_3	K_4	K_5	K_6
$\lvert E \rvert \to$	0	1	3	6 (1+2+3)	10 = $\frac{4(5)}{2}$ (4+3+2+1)	15 = $\frac{5 \cdot 6}{2}$ (1+2+3+4+5)

Circle C_n $\quad n \geq 3$



C_3 $\quad C_4$ $\quad C_5$

Wheel W_n is C_n plus one axle



W_3 $\quad W_4$ $\quad W_5$

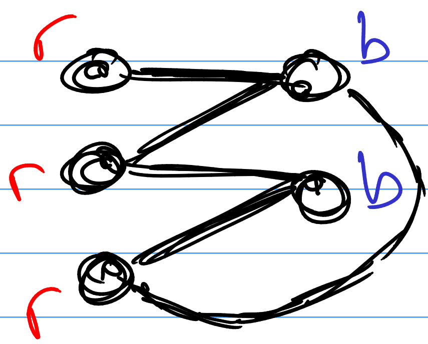
bipartite graphs

$G = (V, E)$

and $V = V_1 \cup V_2$ partition.

and all edges only connect to/from V_1, V_2

(Ex)



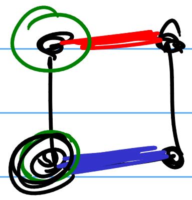
coloring th^n: if you can color vertices in only two colors where every edge connects opposite colors

→ G is bipartite.

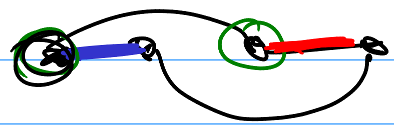
"Same" graphs?

Isomorphism

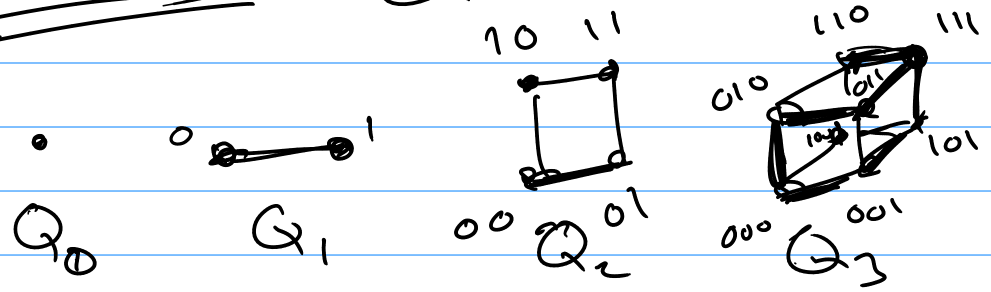
Isomorphic

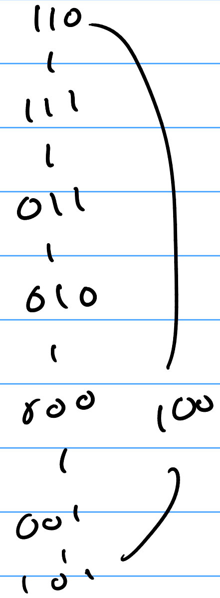


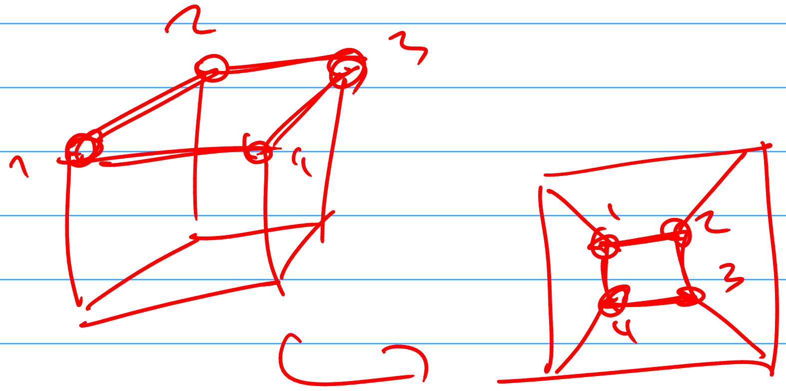
C_4



n-Cubes Q_n







Isomorphic?

Not isomorphic:

1. different number of vertices
2. different number of edges
3. degrees are not preserved
4. paths are not preserved.

How to show isomorphic?

Representing Graphs

① $G = (V, E)$ use lists

② edge directory

$E = \{(1,1), (1,2), (2,2), (2,3), (2,4)\}$

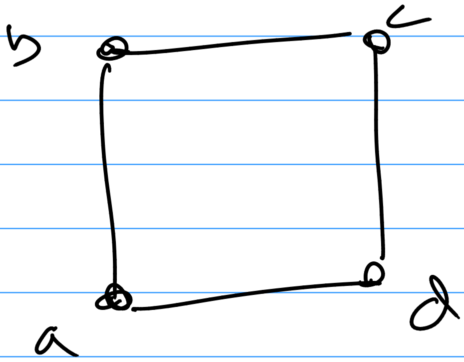
directory [1: [1,2], 2: [2,3,4]]

③ adjacency matrix

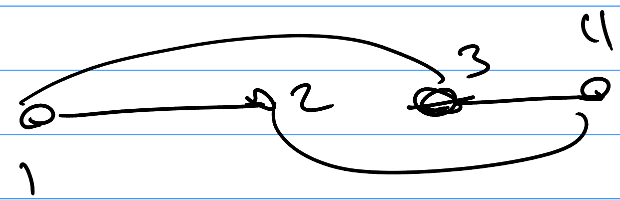
$$M_G = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

to show isomorphic

G_1



G_2



bijection

a → 1
d → 2
b → 3
c → 4

$$A_{G_1} = \begin{array}{c|c} & a\ d\ b\ c \\ \hline a, d, b, c & [\ ?\ ] \end{array} \qquad A_{G_2} = \begin{array}{c|c} & 1\ 2\ 3\ 4 \\ \hline 1, 2, 3, 4 & [\ ?\ ] \end{array}$$