Math 322

represent

Graphs → ① list $G = (V, E)$

② Image

③ - Adj. Matrices
- Adj. directory

Connectivity

① (connected) vs Disconnected

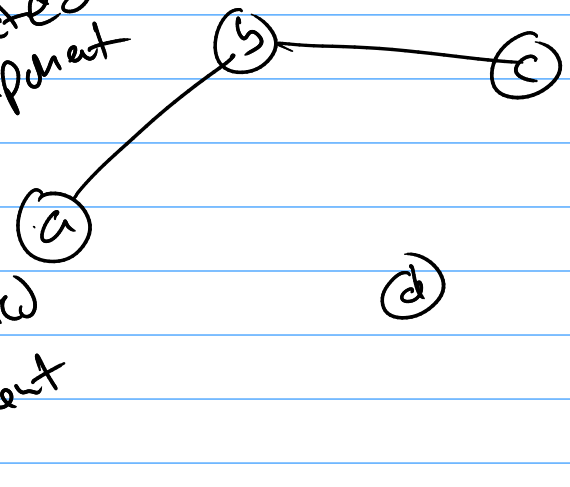
paths between vertices

no paths between vertices

undirected

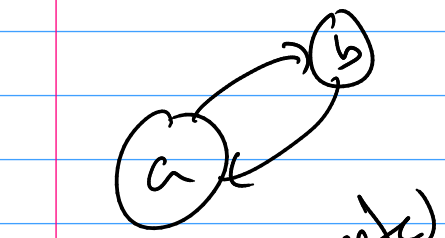
connected: 1 connected component

disconnected: more than one connected component



Directed Graphs:

For paths, path to and from v_i, v_j



Strongly connected

connected or weakly connected

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get $S^+ = \underbrace{S + S^2 + \dots + S^{|V|}}_{\text{all path connectors}}$

undirected graph if S^+ has a 0 in it → 0 is no path so not connected.

$S^+ = [c_{ij}]$

if all $c_{ij} \neq 0$ → connected

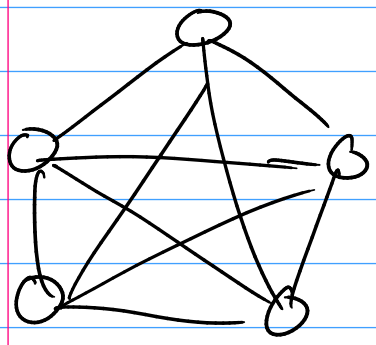
Directed graph

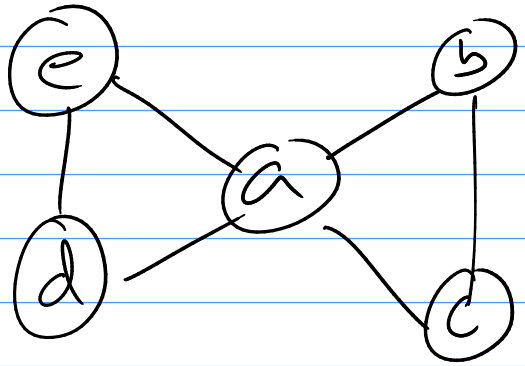
① if all $c_{ij} \neq 0$ → strongly connected

② if $c_{ij} \neq 0$ or $c_{ji} \neq 0$ $i \neq j$

→ (connected or weakly connected)

cut edges / cut vertices





K_5

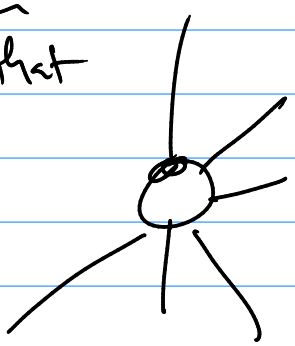
Euler / Hamilton Problems

① Euler Problems

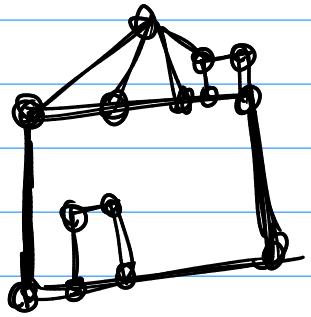
given a graph $G=(V,E)$

a) Does a simple circuit that uses <u>all</u> edges exist?

b) (if no) Does at least a simple path (not circuit) that uses <u>all</u> edges exist?

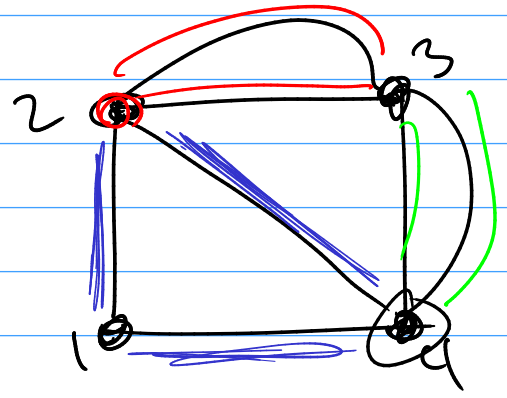


3D additive printing



Thm $\deg(v)$ for all $v \in V$ is even

iff Euler Circuit exists.



1, 2, 4, 1

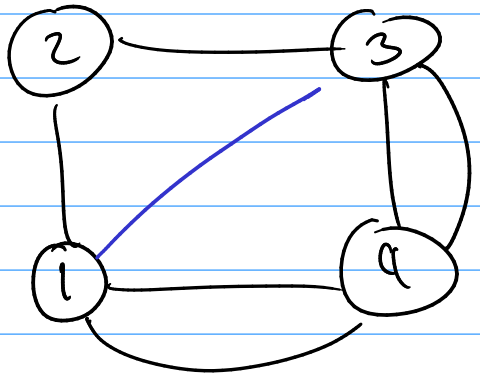
1, 2, 3, 2, 4, 1

1, 2, 3, 2, 4, 3, 4, 1

1
2
4
3
3
4
2

Thm exactly two vertices have odd degree and all other are even

iff Euler Path exists

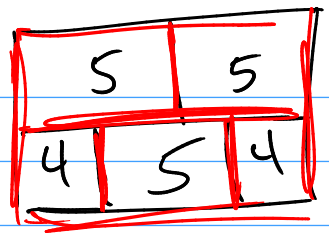


2, 3, 4, 3, 1, 4, 1, 2

2
3
1
4
4
1 ≠ 3

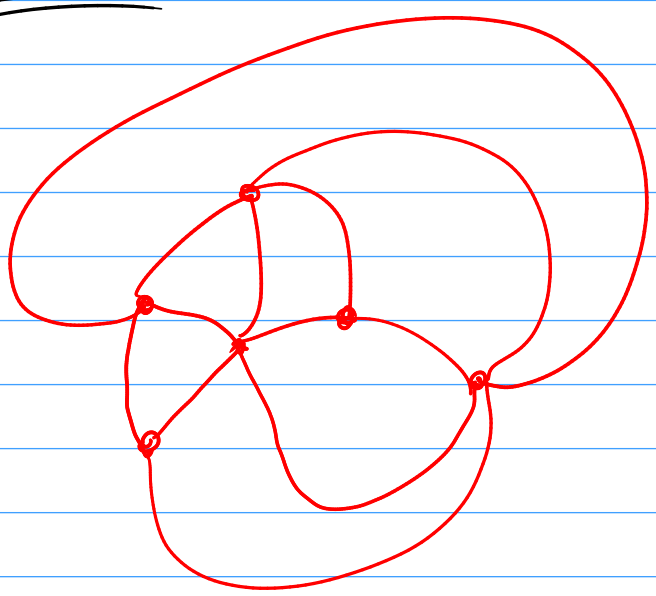
1, 4, 1, 2, 3, 4, 3

9



Cut puzzle

Map problem



Hamilton Problems

Simple circuits/paths that visit every vertex exactly once.

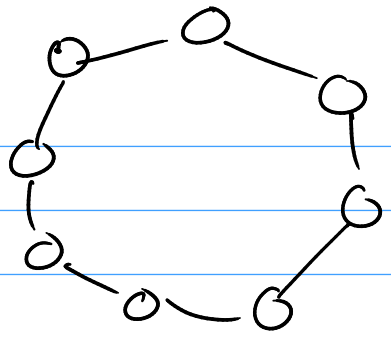
(Note: except on circuit it is ok to start (end) at same vertex)

all graphs

graphs with Hamilton circuits

Thm $\deg(v) \geq \frac{|V|}{2}$ **then** Hamilton circuit exists

(Ex)



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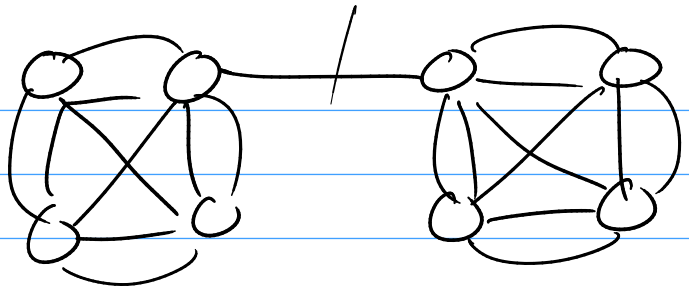
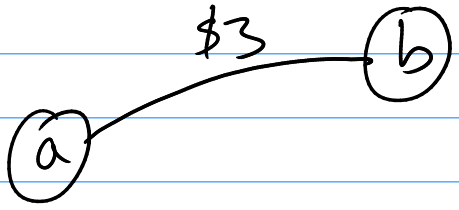


9.5 Weighted Graphs

weight function

$G = (V, E, \omega(e))$



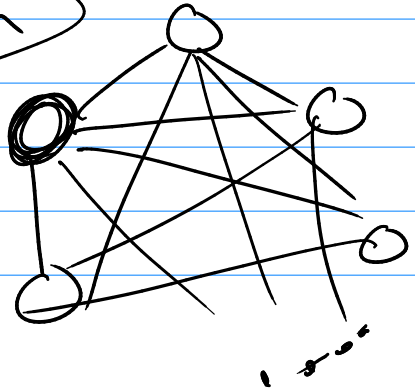
length (path) = Sum of weights of its edges

Applications

1. Minimal / maximal paths (Sing)

(Ex) Traveling Salesman

- n - cities
- Know cost between any two cities



#1 you have $(n-1)!$ Hamilton circuits

#2 each has a cost

#3 least cost?