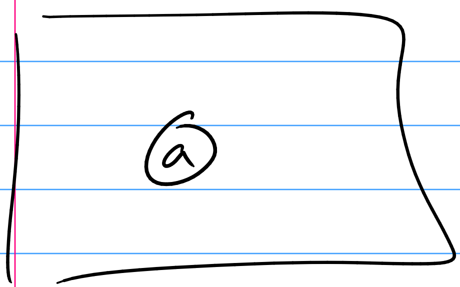
Math 322

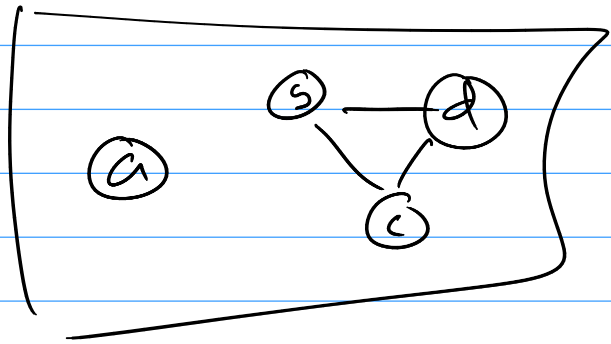
Q's **Connected?** Concept is "the **piece**"

G_1



vs

G_1



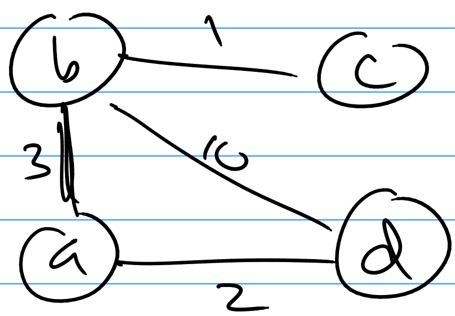
2 connected components

Up Next: 9.5 into ch 10 | **trees**

Weighted graphs: $G = (V, E, w(e))$

↑ weight function

ex



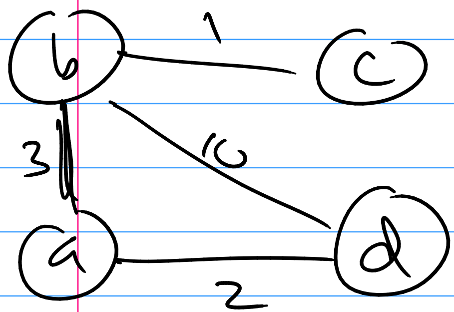
$w(\{a,b\}) = 3$

$w(\{a,d\}) = 2$

$w(\{b,c\}) = 1$

$w(\{b,d\}) = 10$

B weighted graph length path is sum of weights in path.



path: a, b, d, b, c, b

before → length = 5 (edges)

Now in a weighted graph

length = 3 + 10 + 10 + 1 + 1
= 25

App Optimization & path lengths

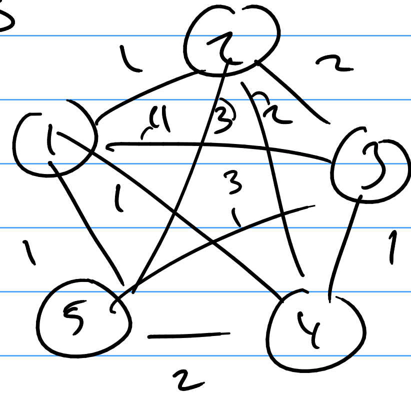
(1) Minimal paths that solve problems

(2) Maximal paths that solve problems

(ex) traveling Salesman problem → $(n-1)!$ unique hamilton circuits.

K_n with weights

(ex) 5 city



So 1, 2, 3, 4, 5, 1 length = 7

or 1, 3, 5, 2, 4, 1 length = 11

⋮

but 4! hamilton circuits is finding 4·3·2·1 = 24 circuits and their costs.

Min = Min

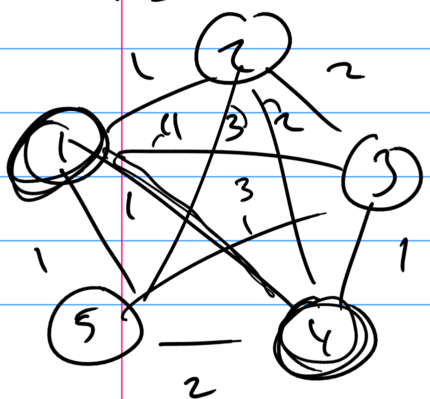
Max = Max

Approx Ans? Heuristic Algorithms

Analysis of Heuristic Algorithms

1. limit the problem by limiting understanding of V, E, wt(e)
2. then approx ≈ (const) · optimal

Ex closest neighbor algorithm

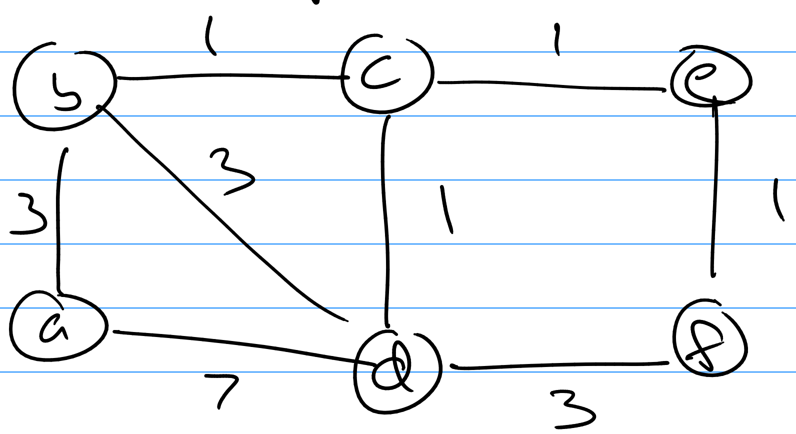


closest neighbor 1, 4, 3, 5, 2, 1

1 1 1 3 1

length = 7

given $G=(V,E,w(e))$ can you find the list of all paths starting at v_i and list each other $v_j \neq v_i$ as a cheapest path?



simple.

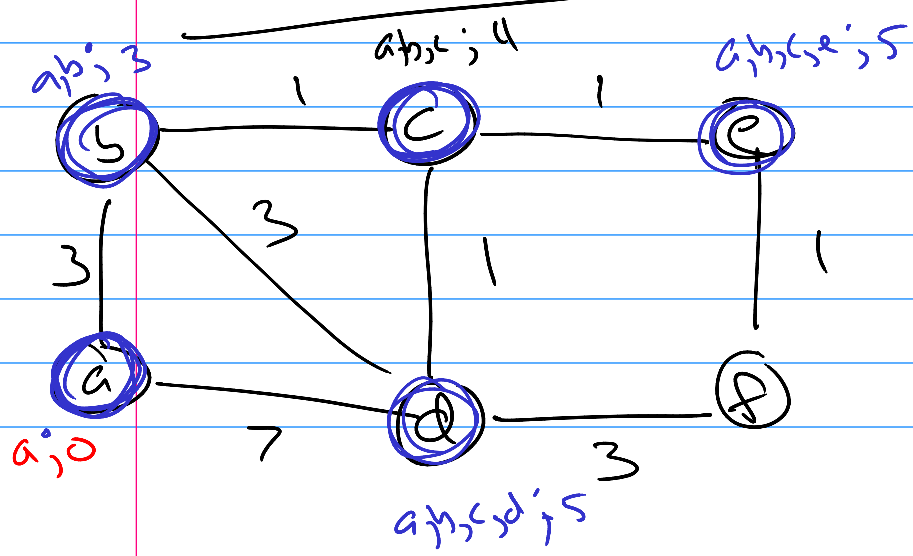
Start @ (a) → a to a ⇒ cheapest paths
a to b
a to c
a to d
a to e
a to f

Scratch
known ⇒ unknown
b,c 4
b,d 7
a,d 7

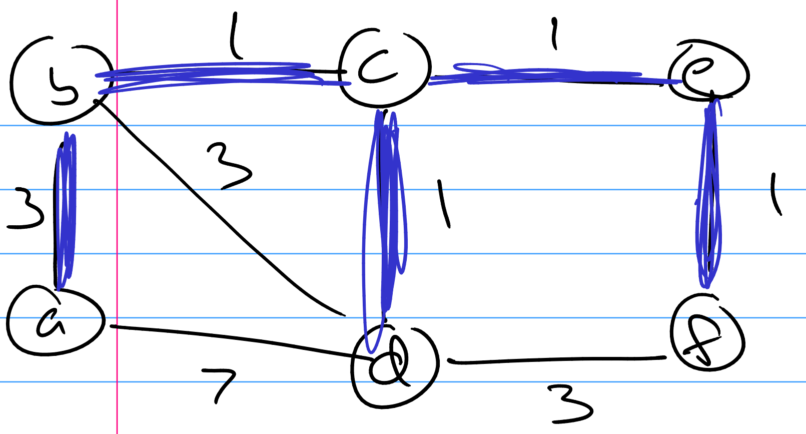
Dijkstra's Algorithm

known: a,b

unknown: c,d,e,f



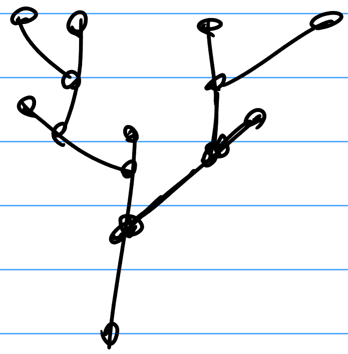
Vertex	Path	Cost
a	a	0
b	a,b	3
c	a,b,c	4
e	a,b,c,e	5
d	a,b,c,d	5
f	a,b,c,e,f	6



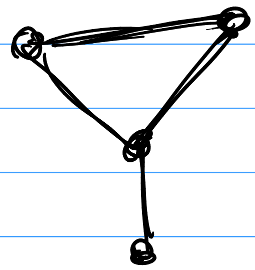
a: 0
a, b: 3
a, b, c: 4
a, b, c, e: 5
a, b, c, d: 5
a, b, c, e, f: 6

Ch 10 — trees

Def: Simple undirected graph with no simple circuits is a tree.



tree



not a tree

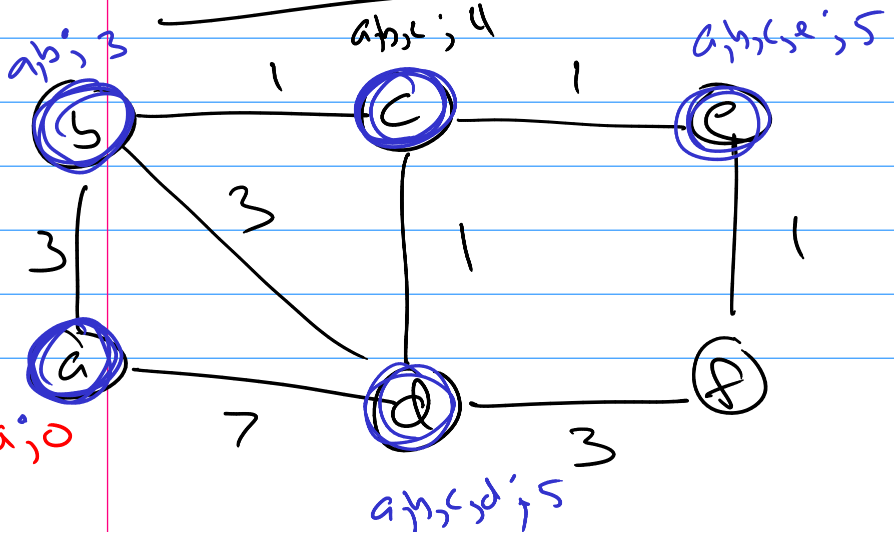
Th^n The following statements are logically equivalent

1. G is a tree (has no simple circuits)
2. A graph is connected and if you remove **any** edge $(V, E - \{e\})$ that graph is disconnected.
3. A graph has no simple circuit, but if you add an edge it will.
4. A graph is connected and $|E| = |V| - 1$
5. Unique simple path between any two vertices.

Pause back dijkstra's

Dijkstra's Algorithm

known	unknown
a, b	c, d, e, f



Vertex	Path	Cost
a	a	0
b	a,b	3
c	a,b,c	4
e	a,b,c,e	5
d	a,b,c,d	5
f	a,b,c,e,f	6

App

list of a to ( )

list of b to ( )

list of c to ( )

⋮

least cost paths

Middle?

Idea

for all least cost paths just count