Math 322 ... Exam 1

① 10 pts / problem

② Partial Credit?

(a) Mark where the Mistakes are

explain

(b) why is it wrong?

how to correct

(c) what to do to correct it?

(d) blank problem?

→ explain why! (+2 pts)

0) Exam Start Time:

0) Name: Key

0) MyWSUid:

0) Sign below to state that you have read and understand the WSU policy on Student Academic Integrity

https://www.wichita.edu/about/policy/ch_02/ch2_17.php

0) Please explain all your answers in the exam.

1) Given the relation $r_1 = \{(a, b)|b = 2a\}$ and $r_2 = \{(a, b)|b = 3a - 1\}$ on the set of positive integers from 1 to 6. Give the list of ordered pairs for r_1 and r_2.

limited numbers

$r_1 = \{(1,2), (2,4), (3,6)\}$

$r_2 = \{(1,2), (2,5)\}$

r_1 $b = 2a$

$1 \to 2(1) = 2$ $(1,2)$

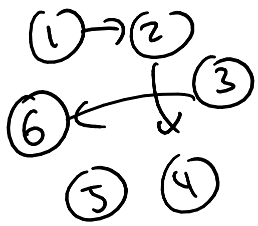
$2 \to 2(2) = 4$ $(2,4)$

$3 \to 2(3) = 6$ $(3,6)$

$4 \to 2(4) = 8 \notin 1$ to 6

2) Using the relations from problem 1 Represent the relation r_1 as a digraph. And represent the composition $r_1 r_2$ as a matrix.

$$M_{r_1} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



$r_1 = \{(1,2),(2,4),(3,6)\}$

$r_2 = \{(1,2),(2,5)\}$

$r_1 r_2 = \{(1,5)\}$

$$M_{r_1 r_2} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{M_{r_1}} \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{M_{r_2}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

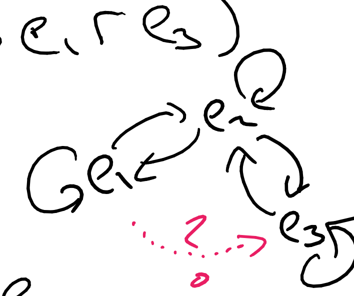
3) Is the relation r consisting of all ordered pairs (a, b) such that a and b are people and have ~~exactly?~~ one common parent: reflexive, symmetric, antisymmetric, and/or transitive? Check all the properties and if a property doesn't hold give a counter-example. Also, **state the logical definitions of the properties** as you consider them.

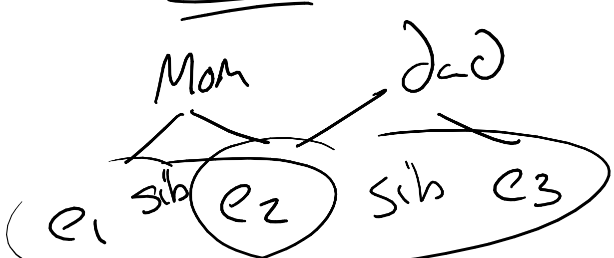
$\boxed{\text{reflexive}}$ $\forall e (eRe)$ is people have one common parent with themselves is $\boxed{\text{true}}$

$\boxed{\text{symmetric}}$ $\forall e_1 e_2 (e_1 R e_2 \to e_2 R e_1)$ is $\boxed{\text{true}}$ b/c the statement "one common parent" is commutable,

$\boxed{\text{antisymmetric}}$ $\forall e_1 e_2 (e_1 r e_2 \land e_2 r e_1 \to e_1 = e_2)$ is $\boxed{\text{False}}$. counter example: siblings

$\boxed{\text{transitive}}$ $\forall e_1 e_2 e_3 (e_1 r e_2 \land e_2 r e_3 \to e_1 r e_3)$ is $\boxed{\text{False}}$ counter example...





but $e_1 \not r e_3$

$f' = g'$

4) Show that the relation r consisting of all pairs of functions (f, g) such that the first derivative of f and the first derivative of g are equal is an equivalence relation on the set of all polynomials with real-valued coefficients. Then find the functions that are in the same equivalence class as $f(x) = x^2 + x + 1$?

ref? sym? transitive?

properties of equality of functions

(a) reflexive: is $\boxed{\text{true}}$ b/c $f' = f'$

symmetric is $\boxed{\text{true}}$ b/c if $f' = g'$ then $g' = f'$

transitive is $\boxed{\text{true}}$ b/c if $f' = g'$ and $g' = h'$ then $f' = h'$

so $\boxed{\text{equiv. relation}}$

$f' = g'$

(b) equiv class of $f(x) = x^2 + x + 1$ is all functions g such that $\boxed{f\,r\,g}$

$f' = 2x + 1$ so $\boxed{g = x^2 + x + C}$

so $g' = 2x + 1 \rightarrow \int g'\,dx = \int (2x+1)\,dx$

$= x^2 + x + C$

equiv class $= [f] = \{ g(x) \mid g(x) = x^2 + x + C \}$
$C \in \mathbb{R}$

5) Show that the divides operator on the set of positive integers is a partial ordering.

① reflexive?
② anti symmetric?
③ transitive?

reflexive: is $\boxed{\text{true}}$

b/c $a \mid a$ is true

w/ $a \cdot 1 = a$.

anti symmetric is $\boxed{\text{true}}$ or $a \cdot k_1 \mid a$ so $k_1 = 1$

b/c $a \mid b \wedge b \mid a$ is $a \cdot k_1 = b \wedge b \cdot k_2 = a$

so $b \cdot k_1 \cdot k_2 = b$, so $k_1 \cdot k_2 = 1$

so $k_1 = k_2 = 1$ (int. soln's)

$\therefore$ $a = b$

$a \mid b \wedge a \neq b \rightarrow b \nmid a$

$a \mid b \wedge b \mid a \rightarrow a = b$

transitive: is $\boxed{\text{true}}$

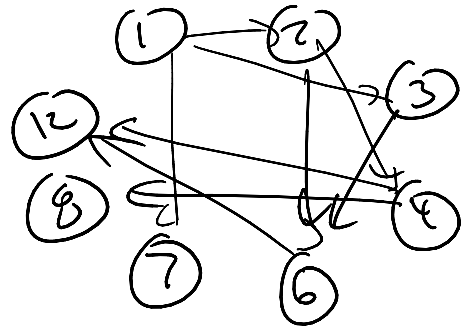
b/c $a \mid b \wedge b \mid c$ is $a \cdot k_1 = b$ & $b \cdot k_2 = c$

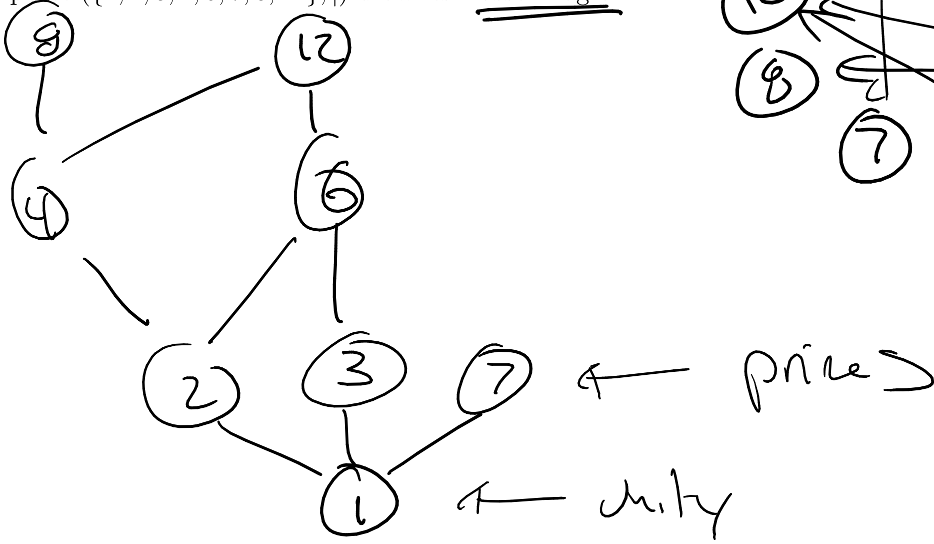
or $a \cdot k_1 \cdot k_2 = c$ so $a \mid c$

$\therefore$ partial ordering

6) For the poset $(\{1, 2, 3, 4, 6, 7, 8, 12\}, |)$ draw its Hasse Diagram.

divides



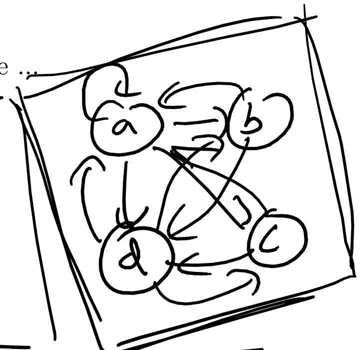


7) For $R = \{(a, a), (a, c), (a, d), (b, a), (b, d), (c, a), (c, d), (d, a), (d, c)\}$ on the set $A = \{a, b, c, d\}$ find the ...

a) Reflexive Closure as a matrix.

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{matrix} a & b & c & d \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\boxed{M_{R \cup I} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}}$$

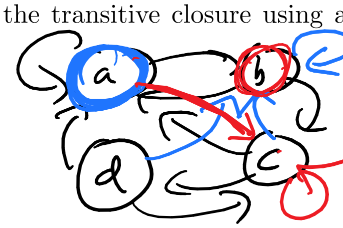


b) Symmetric Closure as a digraph.

$$M_{R \cup R^{-1}} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}}$$

8) For $R = \{(a, a), (a, b), (b, a), (b, c), (c, a), (c, d), (d, a), (d, c)\}$ on the set $A = \{a, b, c, d\}$ find the transitive closure using any method.

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \qquad W_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$



$$W_2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \qquad \boxed{W_3 = W_4 = M_{R^+} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}}$$

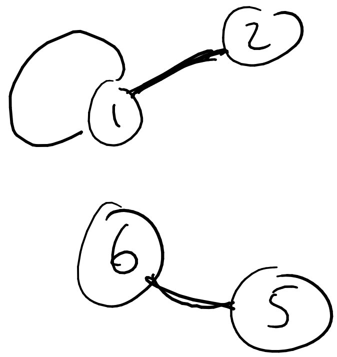
or $R + R^2 + R^3 + R^4 = R^+$

or $R_1 = R$
$R_2 = R_1(I + R_1)$
$R_4 = R_2(I + R_2)$

loops and multiple edges allowed.

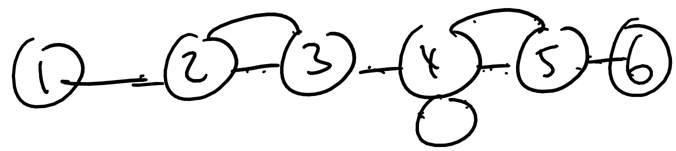
9a) Draw a pseudograph with 6 vertices each with odd degree.

3pts



disconnected

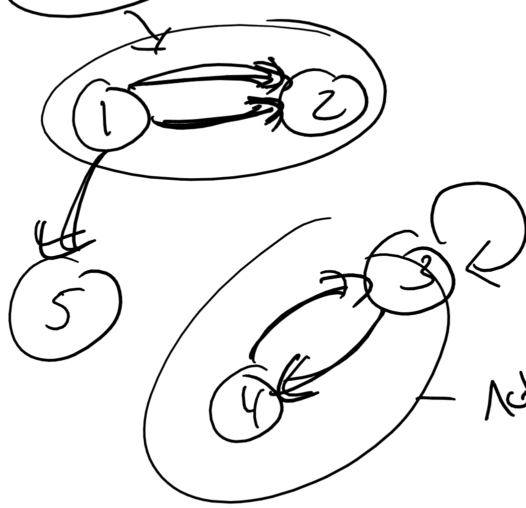
connected



deg seq = $\{5, 3, 3, 3, 1, 1\}$

3pts

9b) Draw a directed multigraph with 5 vertices. One vertex should have only out-degree.



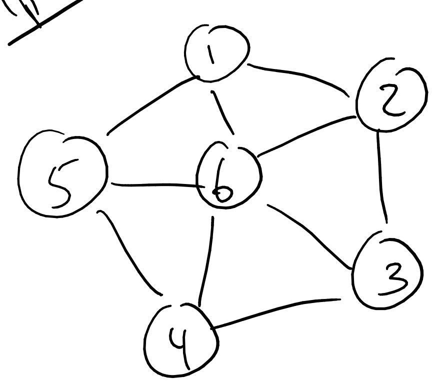
(1) is all out degree

not multiple edges

C_5 with center vertex

9c) Draw the graph W_5, label each vertex, and state the number of vertices, edges, and degree for each vertex.

9pts



$|V| = 6$

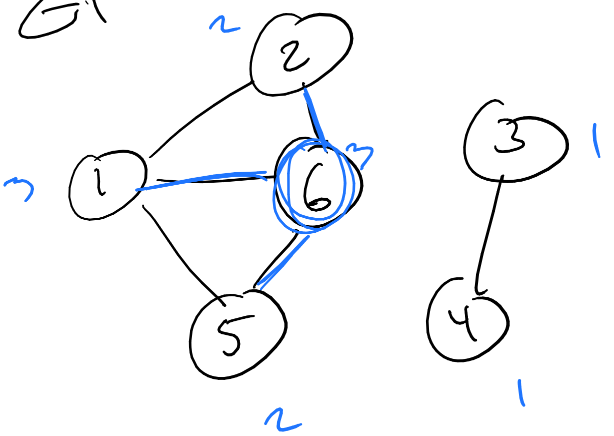
$|E| = 10$

$\deg(1) = \deg(2) = \deg(3) = \deg(4) = \deg(5) = 3$

$\deg(6) = 5$

10) Draw two graphs that are NOT isomorphic. But they both have six vertices, the same number of edges (your choice on how many), and the same degree sequences (your choice). Show why your two graphs are NOT isomorphic.

G_1

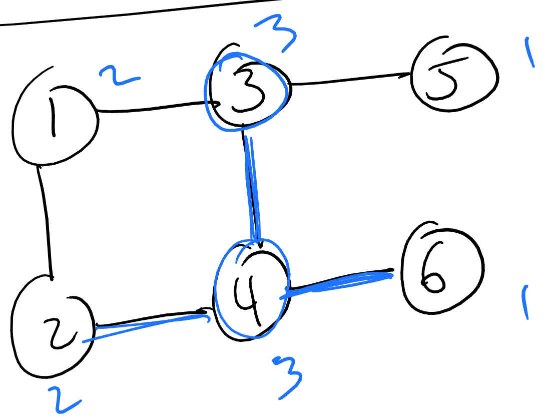


$|V| = 6$

$|E| = 6$

degrees $= \{3, 3, 2, 2, 1, 1\}$

G_2



$|V| = 6$

$|E| = 6$

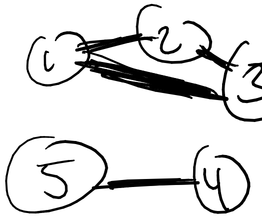
degrees $= \{3, 3, 2, 2, 1, 1\}$

Not isomorphic b/c G_1 is disconnected but G_2 is connected.

{1, 3}

5pts

11a) For graph G = (V,E) ~~with vertices~~ $V = \{1,2,3,4,5\}$ and undirected edges $E = \{\{1,2\},\{1,3\},\{2,3\},\{4,5\}\}$ represent G as a matrix. Is the graph connected? If the graph is not connected what are the connected components?



$$M_G = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

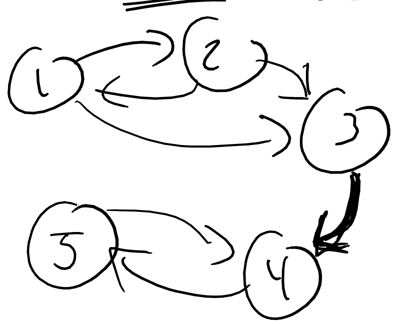
$$M_{G^T} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

check

Not connected: Connected Components are {1, 2, 3} and {4, 5}

5pts

11b) For the graph G = (V,E) with vertices $V = \{1,2,3,4,5\}$ and directed edges $E = \{(1,2),(1,3),(2,1),(2,3),(3,4),(4,5),(5,4)\}$ represent G as a digraph. Is the graph strongly connected? Is the graph weakly connected?



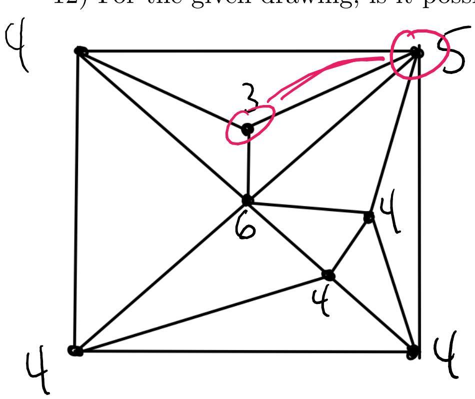
a) not strongly connected b/c no paths from 4 to 1, 2, or 3

b) weakly connected b/c ignoring direction it is connected.

$$M_R = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$M_{R^+} = [\ ?\ ]$

12) For the given drawing, is it possible to draw it without picking up your pencil? Explain your answer.



Yes. exactly two vertices of odd degree ∴ Euler Path exists

path starts/ends @ vertices of deg 3 and 5.

0) What is the time you ended working on the exam and started scanning it?

to make euler path, add red edge

Make euler circuit and cut added edge

Trees Continued

Thm's $\quad n = |V|$

(1) full-mary $\quad \left[\begin{array}{l} n = mi + 1 \\ n = i + l \end{array}\right.$

So given i or l or n we can find other two.

(2) $l \leq m^h$

or

(3) $h \geq \lceil \log_m l \rceil$

and $h = \lceil \log_m l \rceil$ if full and balanced

Apps?

~~(1) Decision Trees~~

$e_1 r e_2 \land e_2 r e_1 \to e_1 = e_2$

Proof?

infinite

~~eg~~

elms = {1, 2, 3, 4, (- -) }

examples!

$\cancel{2|6 \to 6 \nmid 2}$

$\cancel{4}$

$6|6$

$2|6$

$$\boxed{e_1 | e_2 \land e_2 | e_1 \to e_1 = e_2}$$

if e_1 is a factor of e_2 ~~&~~ is

e_1 is e_2 or e_1 smaller e_2

and e_2 is a factor of e_1

is e_1 is e_2 or e_2 smaller e_1