Math 322

Q's 10.1 #3 (proof by induction)

basis? inductive step?

Thm G is a simple undirected graph with no self loops

connected, no loops → $\boxed{G \text{ is a tree}}$ **iff** G connected and $|E| = |V| - 1$

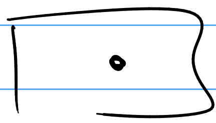
Note: Induction? P(n) is true for $n = 1, 2, 3, 4, \ldots$

given a graph $G = (V, E)$

then $|V| = n = 1$ or 2 or 3 or 4 or ...

P(n)? $|E| = |V| - 1$ true or false

Thm G is a tree iff connected an for $|V| = n$
$|E| = |V| - 1$
$n = 1, 2, 3, 4, \ldots$

Pf **Basis:** $(n = 1)$ is G = 

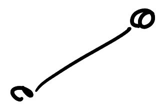
$|V| = 1$ $|E| = 0$ **true**

$|E| \overset{?}{=} |V| - 1$ is $0 \overset{?}{=} 1 - 1$

Play Scratch

$n=2$

$|V|=2$

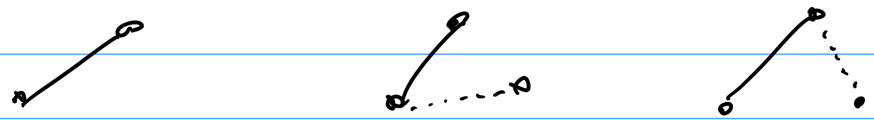


$|V|=2 \quad |E|=1$

$\therefore |E| \overset{?}{=} |V|-1$

$1 \overset{?}{=} 2-1$ **true**

$n=2 \xrightarrow{?} n=3$



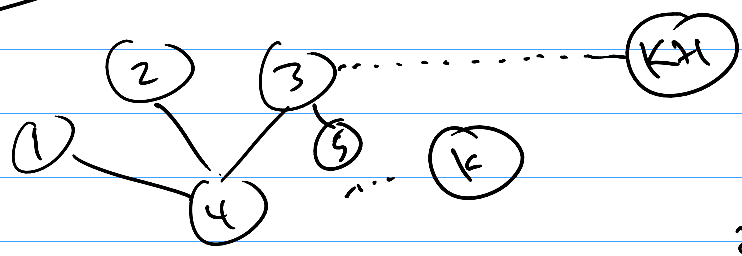
$|E|=2 = |V|-1$ $\qquad$ $|E|=2 = |V|-1$

Inductive: Assume for K vertices that $|E| = |V|-1$

$|E| = K-1$

now what about $K+1$ vertizes?

Start with tree of K vertizes. with its $K-1$ edges



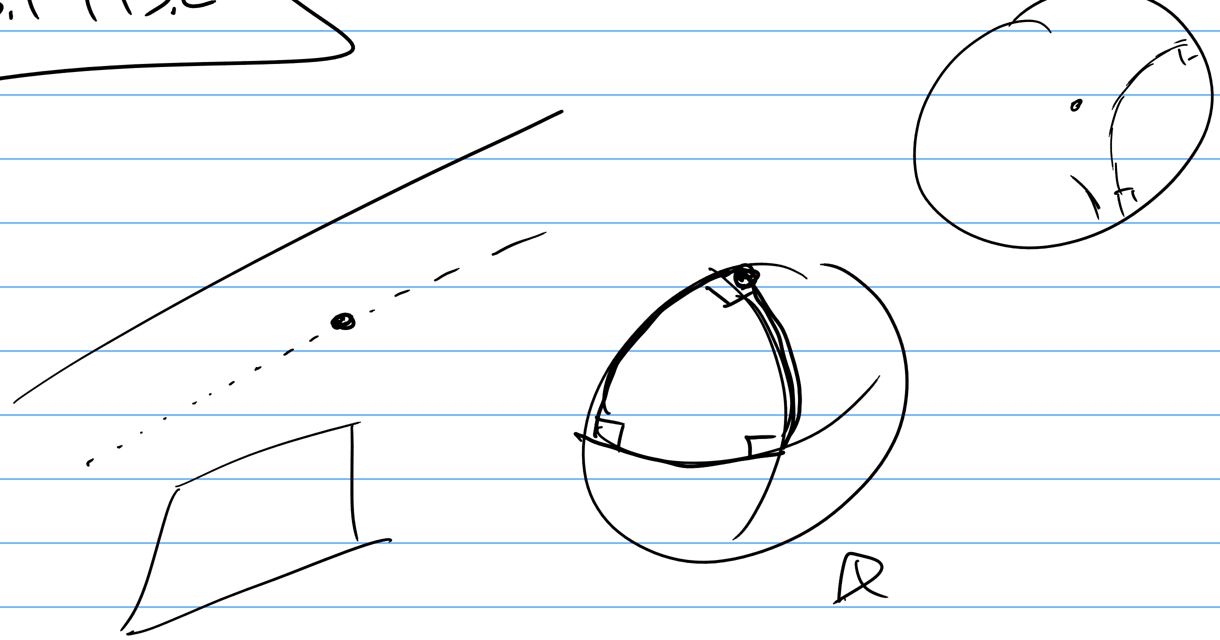
connect the $K+1$ vertex with one edge to the tree of K.

$\rightarrow$ Now: K edges!

so $|E| = k$ $\quad |V| = k+1$

so $k = (k+1) - 1$ $\boxed{\text{true}}$

13.1 | 13.2



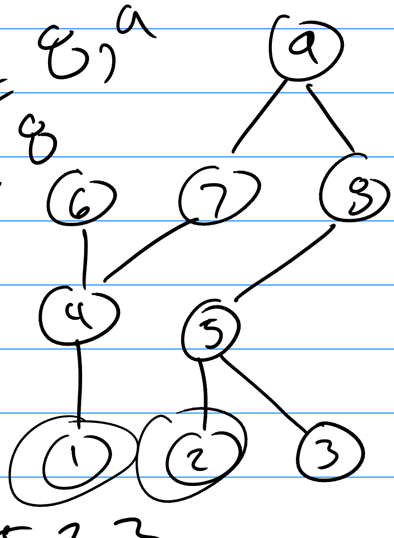
Posets

Set S with a relation that is reflexive, antisym, transitive.

Hasse Diagram

(upper bounds of 5, 8) = 8, 9

(least of 5, 8's upper bounds) = 8



Maximals: 6, 9

Minimals: 1, 2, 3

Greatest: None — the $\boxed{\text{one}}$ maximal

Least: none — the $\boxed{\text{one}}$ minimal

(lower bounds of 5, 8) = 5, 2, 3

(greatest of 5, 8's lower bounds) = 5

if we focus on pairs then we can

consider greatest of lower bounds of a, b is a function, call this the <u>meet</u>

$$= \boxed{a \wedge b}$$

consider least of upper bounds of a, b is a function, call this the <u>join</u>

$$= \boxed{a \vee b}$$

thm <u>13.1.6</u> if the least upper and greatest lower bounds exist, then they are unique.

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restrict Poset such that any pair a, b has a <u>meet</u> and <u>join</u>

call the Poset a **Lattice**

<u>Now:</u> $[L; \wedge, \vee]$ is an <u>Algebraic System</u>

Lattice – Poset – set of objects with a relation that is ref, antisym, trans.

<u>Concept:</u> $a \wedge b$ = greatest lower (shared below)

$a \vee b$ = least upper (first thing above that both make)

Note: we can prove that $\wedge, \vee$ are

Laws

1. Commutative
2. associative
3. idempotent
4. absorption

← all lattices

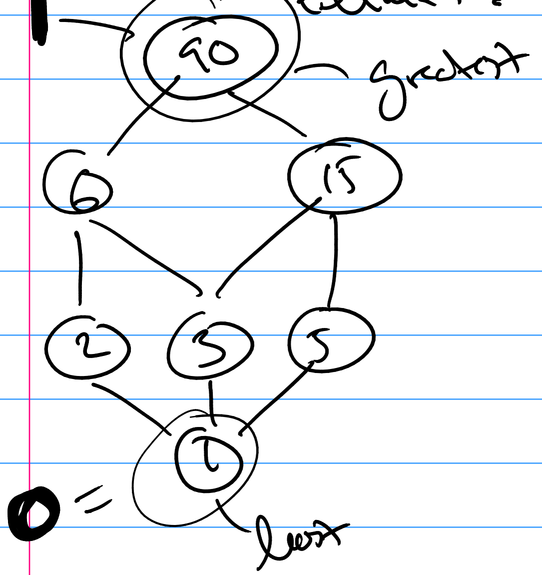
??. Distributive ??. $a \wedge (b \vee c)$

Not for all lattices

⇒ Restrict to only Lattices that are also distributive.

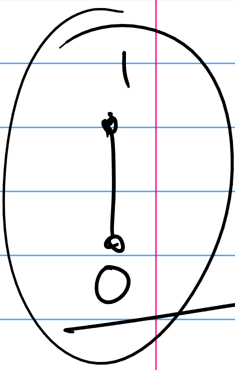
Call them: $\boxed{\text{Distributive Lattices}}$

⇒ Now Restrict these to Distributive Lattices that they have a least and greatest element.



→ $\boxed{\text{Bounded Lattice}}$

Complemented Lattice



$$a \vee b = 1$$
$$a \wedge b = 0$$

call $b = \overline{a}$