Math 322

13.6 → 13.4 → 13.5

C.S. Boolean Algebra $B = \{0,1\}$ ← element

x	y	$x \wedge y$	$x \vee y$	$\bar{x}$	$\bar{y}$
1	1	1	1	0	0
1	0	0	1	0	1
0	1	0	1	1	0
0	0	0	0	1	1

← 3 operations

$\wedge, \vee$

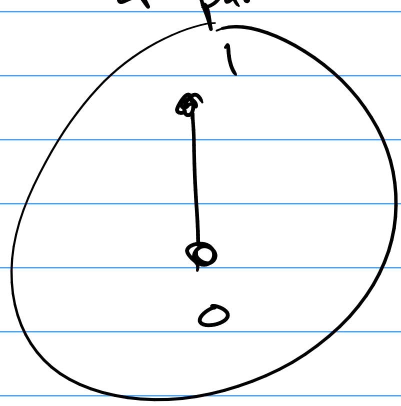
$\overline{}$

given operators exist and are these definitions

we get (show) the laws.

vs ch 13 Boolean Algebra ← from the study of partial orders

we added restrictions till we got the the simplest

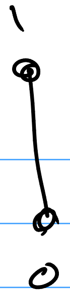


why is

$1 \wedge 1 = 1$
$1 \wedge 0 = 0$
$0 \wedge 1 = 0$
$0 \wedge 0 = 0$?

why is

$0 \vee 0 = 0$
$0 \vee 1 = 1$
$1 \vee 0 = 1$
$1 \vee 1 = 1$?

So  is our boolean algebra $\{B_2; \wedge, \vee, \bar{\ }\}$

Meet ≡ greatest lower bound

Join ≡ least upper bound

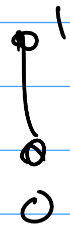
negation = complement

$1 \wedge 1 = 1$
$1 \wedge 0 = 0$
$0 \wedge 1 = 0$
$0 \wedge 0 = 0$

$1 \vee 1 = 1$
$0 \vee 1 = 1$
$1 \vee 0 = 1$
$0 \vee 0 = 0$

$\bar{0} = 1$

$\bar{1} = 0$

Boolean Algebra & only  $[B_2 ; \wedge, \vee, \bar{\ }]$

$B_2 = \{0, 1\}$ .. what can we do?

Algebra vs Arithmetic

Algebra: Introduce an unknown object. (Variable)

Arithmetic: do ops on known objects

x is 0 or 1 y is 0 or 1

Q I know $\boxed{x \wedge y = 1}$ what is x? what is y?

because under $\wedge$ .. $1 \wedge 1 = 1$ is the only outcome

so $\boxed{x = 1 \text{ and } y = 1}$

~~Mix~~ variables and operators and elements

→ called an expression

$$(x \wedge 0) \vee \left[\bar{y} \wedge (x \vee y)\right]$$

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x	y	$x \wedge y$	$x \vee y$	$\bar{x}$	$\bar{y}$
1	1	1	1	0	0
1	0	0	1	0	1
0	1	0	1	1	0
0	0	0	0	1	1

0	x	y	$x \wedge 0$	$\bar{y}$	$x \vee y$	$\bar{y} \wedge (x \vee y)$	$(x \wedge 0) \vee [\bar{y} \wedge (x \vee y)]$
0	1	1	0	0	1	0	0
0	1	0	0	1	1	1	1
0	0	1	0	0	1	0	0
0	0	0	0	1	0	0	0

So expressions have input → output pairings

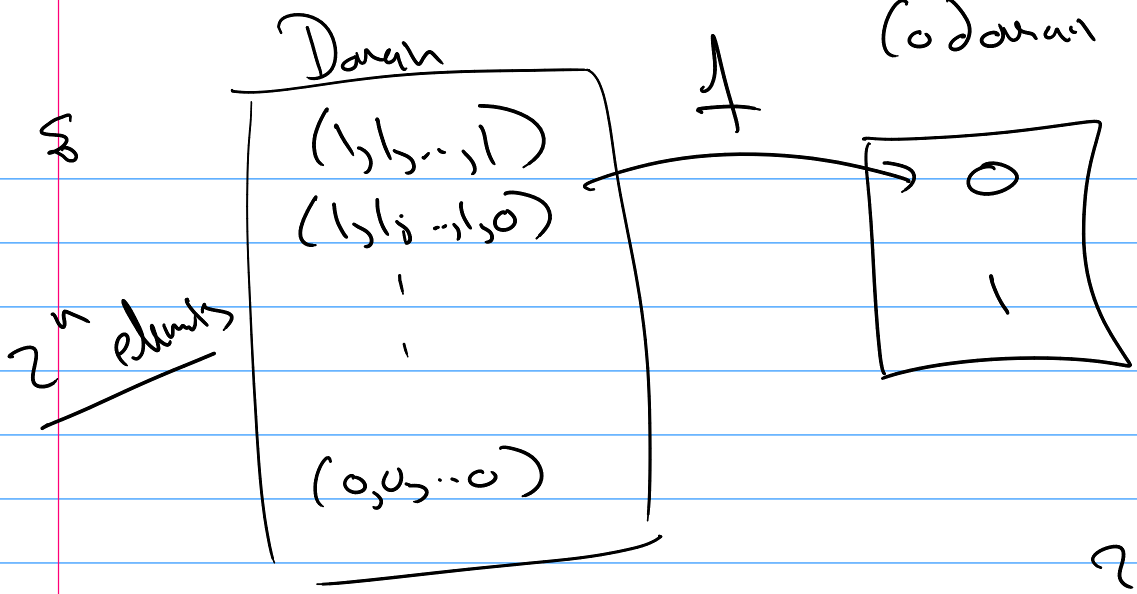
if you have 2 variable you have 4 inputs

has only a 0 or 1 for <u>each</u> input

So expressions are functions based on the number of variables.

if n variable → get 2^n rows = 2^n inputs
→ each input goes to 0 or 1

$\boxed{\text{Boolean function}}$



$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

How many functions are there?

$$|F| = \underbrace{2 \cdot 2 \cdot 2 \cdots 2}_{2^n} = 2^{2^n}$$

Finite!

So

x	y	z	F = f(x,y,z)
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

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writing Boolean functions using minterms or maxterms

Minterms

	x	y	$x \wedge y$	$x \wedge \bar{y}$	$\bar{x} \wedge y$	$\bar{x} \wedge \bar{y}$
$xy \to$	1	1	1	0	0	0
$x\bar{y} \to$	1	0	0	1	0	0
$\bar{x}y \to$	0	1	0	0	1	0
$\bar{x}\bar{y} \to$	0	0	0	0	0	1

what happens if you add (or) minterms?

x	y	$x \land y$	$x \land \bar{y}$	$\bar{x} \land y$	$\bar{x} \land \bar{y}$	F
→ 1	1	1	0	0	0	0
→ 1	0	0	1	0	0	1
→ 0	1	0	0	1	0	1
→ 0	0	0	0	0	1	0

$F = x\bar{y} + \bar{x}y$

↑ plus, times

$F = (x \land \bar{y}) \lor (\bar{x} \land y)$

↑ and / or notation

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x	y	z	F
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

$F = x\bar{y}z + x\bar{y}\bar{z} + \bar{x}\bar{y}\bar{z}$

(minterm expansion

(disjunctive normal form

(sum & products

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x	y	z	F
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	0
0	0	0	1

Maxterm

vs	x	y	$x \lor y$	$x \lor \bar{y}$	$\bar{x} \lor y$	$\bar{x} \lor \bar{y}$
$\bar{x}+\bar{y}$ →	1	1	1	1	1	0
$\bar{x}+y$ →	1	0	1	1	0	1
$x+\bar{y}$ →	0	1	1	0	1	1
$x+y$ →	0	0	0	1	1	1

$F = (x+\bar{y}+\bar{z})(x+y+\bar{z})$

(maxterm expansion

(conjunctive normal form

(product & sums

$x+y$ vs xy

maxterm: focus on 0

minterm: focus on 1

Any function table has a minterm expansion
and a maxterm expansion

which use only $\wedge, \vee, \overline{}$ so
that set $\{\wedge, \vee, \overline{}\}$ operations
are called functionally <u>complete</u>

<u>but</u> we have <u>De Morgan's</u>

$$\overline{xy} = \bar{x} + \bar{y}$$

Double Complement law

$$xy = \overline{\overline{xy}} = \overline{\bar{x} + \bar{y}}$$