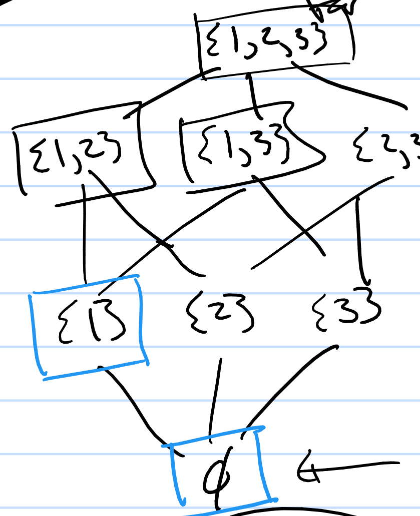
Math 322 13.4 (1ab, 2ab) 13.5 (1, 2) 13.6 (1c & do both min/max terms)

Q's

13.2 example operations $\cap, \cup, ^c$

for $P(A)$ $A = \{1, 2, 3\}$, $\subseteq$ is partial ordering



← choose 2

← choose 1

← choose 0

elmts: 8 of them

ϕ
$\{1\}, \{2\}, \{3\}$
$\{1,2\}, \{1,3\}, \{2,3\}$
$\{1,2,3\}$

operation tables:

Ex Arithmetic

multiplication table

×	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

$P(\{1,2,3\})$

∪	ϕ	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
ϕ	ϕ	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
{1}	{1}							
{2}	{2}							
{3}	{3}							
{1,2}	{1,2}					{1,2,3}		
{1,3}	{1,3}							
{2,3}	{2,3}							
{1,2,3}	{1,2,3}	{1,2,3}	—	—	—	—	—	{1,2,3}

$2^3 = 8^{th}$ order Boolean Algebra

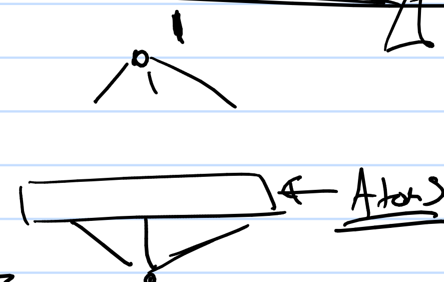
$P(\{1,2,3\})$

$\cap$	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\boxed{\emptyset}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{1\}$								
$\{2\}$								
$\{3\}$								
$\{1,2\}$								
$\{1,3\}$								
$\{2,3\}$								
$\boxed{\{1,2,3\}}$	$\emptyset$	$\{1\}$	$\{2\}$	-	—	—	—	$\{1,2,3\}$

$P(\{1,2,3\})$

	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
c	1, 2, 3	2, 3	1, 3	1, 2	3	2	1	$\emptyset$

13.1 $[B; \wedge, \vee, \bar{\ }]$ poset

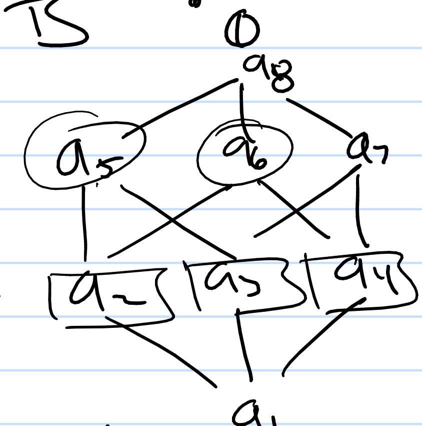


Def: $A = \{a_1, a_2, \ldots, a_n\}$ are atom of B

if $x \wedge a = a$ for all x

or $x \wedge a = \emptyset$

atoms →



Thm for any finite boolean algebra let $A = \{a_1, a_2, \ldots, a_n\}$ the set of all atoms of B. Then every element of B can be expressed uniquely as the join of a subset of A.

$A = \{a_1, a_2, \ldots, a_n\}$ are the atoms of $[B; \vee, \wedge, \bar{\ }]$

Thm ① $[B; \vee, \wedge, \bar{\ }]$ is isomorphic to $[\underline{P(A)}; \cup, \cap, ^c]$

isomorphic means

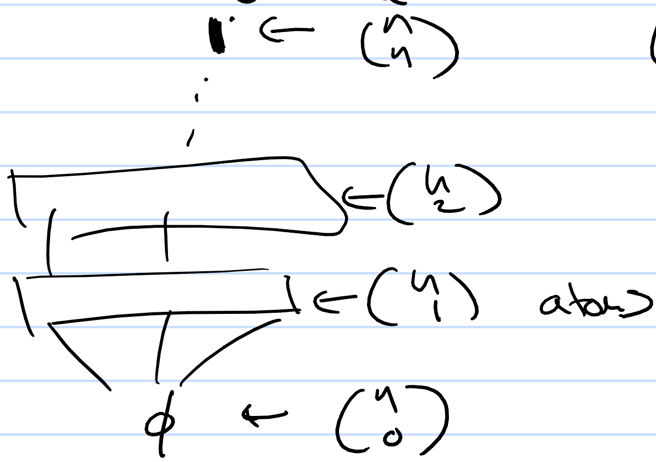
① posets are isomorphic

② $\vee$ of elements in B are $\cup$ of isomorphic elements to $P(A)$

③ $\left.\begin{matrix}\wedge \\ \cap\end{matrix}\right\}$ same as ②

④ complement in B is complement in $P(A)$

② B has 2^n elements.

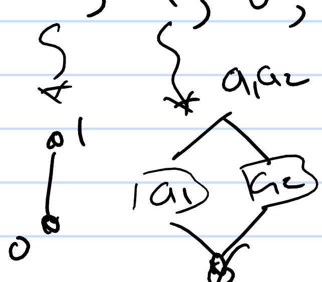


$\binom{n}{0} + \binom{n}{1} + \cdots \binom{n}{n}$

$= \underline{2^n \text{ elements}}$

So for finite Boolean algebras we are order..

$2, 4, 8, 16, \ldots, 2^n, \ldots$



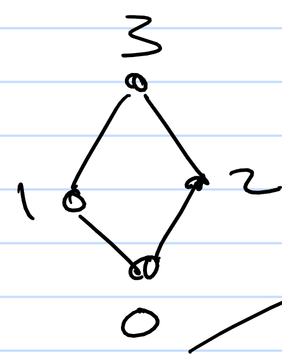
③ for 2^n order type Boolean Algebras..
are all <u>isomorphic</u>.

13.5

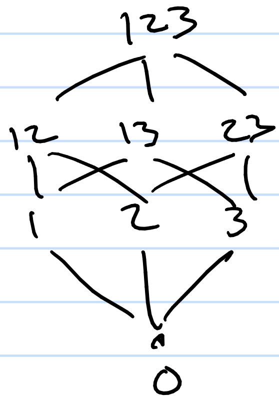
order $2, 4, 8, 16, 32, \ldots, 2^n, \ldots$

order 2

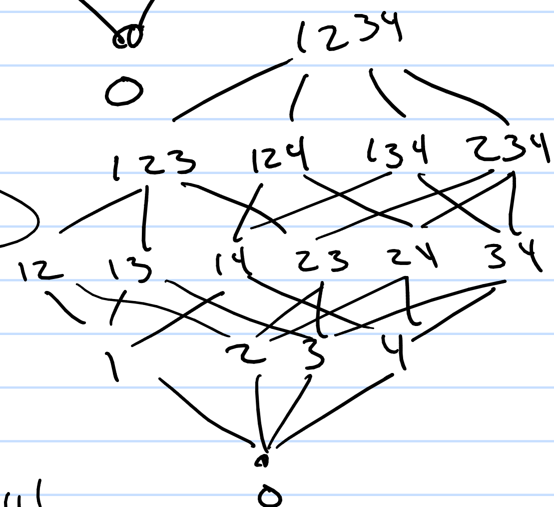
order 4



order 8



order 16



1

1 1

1 2 1

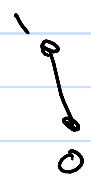
1 3 3 1

1 4 6 4 1

$$\binom{4}{2} = \frac{4!}{2!2!} = \frac{4\cdot 3}{2} = 6$$

Make this easier?

Consider



$\{B_2; \vee, \wedge, -\}$

ops tables

$\vee$	0	1
0	0	1
1	1	1

$\wedge$	0	1
0	0	0
1	0	1

u	$\bar{u}$
0	1
1	0

Consider: $\underbrace{B_2 \times B_2}_{\text{ordered pairs}} = B_2^2$, $\underbrace{B_2 \times B_2 \times B_2}_{\text{ordered triples}} = B_2^3$

(0,0)
(0,1)
(1,0)
(1,1)

(0,0,0)	(1,0,0)
(0,0,1)	(1,0,1)
(0,1,0)	(1,1,0)
(0,1,1)	(1,1,1)

Consider a computer representation of a subset of $\underline{\underline{A}}$

ex $A = \{a_1, a_2\}$

$$\begin{aligned} \emptyset &\rightarrow (0,0) \\ \{a_1\} &\quad (1,0) \\ \{a_2\} &\quad (0,1) \\ \{a_1, a_2\} &\quad (1,1) \end{aligned}$$

$A = \{a_1, a_2, a_3\}$

$$\begin{aligned} \emptyset &= 000 \\ \{a_1\} &= 100 \\ \{a_2\} &= 010 \\ &\vdots \\ \{a_2, a_3\} &= 011 \\ \{a_1, a_2, a_3\} &= 111 \end{aligned}$$

so B_2^n is isomorphic $P(A)$ $|A| = n$ atoms

Def

$x_1 y_1 \vee x_2 y_2$

$= (x_1 \vee x_2), (y_1 \vee y_2)$

$\vee$	00	01	10	11
00	00	01	10	11
01	01	01	11	11
10	10	11	10	11
11	11	11	11	11

$\cup$	$\emptyset$	b	a	ab
$\emptyset$	$\emptyset$	b	a	ab
b	b	b	ab	ab
a	a	ab	a	ab
ab	ab	ab	ab	ab

isomorphic

$x_1 y_1 \wedge x_2 y_2$

$= (x_1 \wedge x_2), (y_1 \wedge y_2)$ also isomorphic

$\overline{x_1 y_1} = \overline{x}, \overline{y}$

u	$\overline{u}$
00	11
01	10
10	01
11	00

u	$\overline{u}$
$\emptyset$	ab
b	a
a	b
ab	$\emptyset$

isomorphic

So All Boolean Algebras of 2^n order

have n atoms, 2^n elements

define them by B_2^n n tuples

define $\wedge, \vee, \overline{\ }$ by element wise ops

$D_{30}, |$ elements $\boxed{1, 2, 3, 5, 6, 10, 15, 30}$

$2 \cdot 3$ $2 \cdot 5$ $3 \cdot 5$ $2 \cdot 3 \cdot 5$

Isomorphism

1 =	000
2 =	100
3 =	010
5 =	001
6 =	110
10 =	101
15 =	011
30 =	111