Math 322

Foundations & Computing

Math.hws.edu/FoundationsOfComputing/

chapters 3 to 5

go ahead and read all 3 chapters.

Computer .1 → .2 → .4 → .8 → 1.6 → .6 → 1.2 → .2 →

$12 = 2 \cdot 2 \cdot 3$

Human Language

→ Natural Human Language

↳ symbols = ideas

 - cat logogram

cat phonogram

rules to make complex ideas from basic ideas

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Formal Languages ← focus on recognition

① Need a set of stuff to build our recognized ideas from.

(Natural Terms)	(Formal terms)
① Vocabulary	① Alphabet
② collection of words	② collection of symbols
③ String together (elements) of the Vocabulary → sentence	③ String together (elements) of the Alphabet → word Sequence or String

Note: ① empty string = string of no length

this is either λ or ε

② Strings or sequences of symbols so their length is the finite number of symbols in them.

$$S = S_1 S_2 S_3 \cdots S_n$$

length of $S = n$

How to Make Strings?

Background

① Size or length is number of symbols in the string.

② Same? a) same length
b) same symbols in same order.

Constructors:

① Concatenation $u = a_1a_2a_3$, $v = b_1b_2b_3$

$u \cdot v = a_1a_2a_3b_1b_2b_3$

② Power: $u^3 = u \cdot u \cdot u$

Note: Inductive definition

$$\begin{cases} u^0 = \varepsilon \\ u^k = u^{k-1} \cdot u \end{cases}$$

③ Union $u = a_1a_2a_3$ $v = b_1b_2b_3$

$u \cup v = u \mid v = \{a_1a_2a_3, b_1b_2b_3\}$

④ reversal $u^R = a_3a_2a_1$

⑤ Kleene Closure u^*

$u^* = u^0 \mid u^1 \mid u^2 \mid u^3 \mid u^4 \mid \ldots$

= set of all strings built from u of any length

$\{ab \mid cb\}^* = \{\varepsilon \mid ab \mid cb \mid abab \mid abcb \mid cbab \mid cbcb \mid \dots\}$

Language = set of strings declared to be in the language.

subset of (Alphabet)* = Σ^*

Σ or V (for vocabulary)

$\Sigma = \{$ set of symbols $\}$ (it is finite)

but $\Sigma^* = \Sigma^0 \mid \Sigma^1 \mid \Sigma^2 \mid \Sigma^3 \mid \dots$

is (countably infinite)

A (language) is just a subset of Σ^*

→ it is one of the subsets of Σ^*

Q: how many (subsets)?

the power set collects all subsets.

$|P(\Sigma^*)| = 2^{|\Sigma^*|} = 2^{\aleph_0} = \aleph_1 = |\mathbb{R}|$

$|\Sigma^*|$: countably infinite, so it's $\aleph_0$

uncountable! Σ