Math 322

Q's c) $L_3 = \{x \mid n_b(x) = 2 \pmod 3\}$ $\Lambda = \{a, b\}$

remainder of 2 under mod 3

$n_b(x)$
↖ how many b's?

$\boxed{n_b(x) = 2}, 5, 8, 11, 14, \ldots$ (+3, +3, +3)

$n_b(ababb) = 3$

$n_b(aca) = 0$

$a^* b a^* b a^*$ 2-b's

"add" $(a^* b a^* b a^* b a^*)^*$

Note: S^* vs S^n $n = ???$

$S^* = \{ S^n \mid n = 0, 1, 2, 3, \ldots \}$

$(a^0 b)^1 a \rightarrow ba \notin n_b() = 2, 5, 8, \ldots$ only

$(a^0 b a^1 b a^0)(b a^* b a^* b a^*)^0$

Note: $(s_1 | s_2)^* \leftarrow$ All strings of s_1, s_2

$\hookrightarrow \varepsilon, s_1, s_2, s_1 s_2, s_1 s_1, s_2 s_1, s_2 s_2, \ldots$

$(0|1)^* \leftarrow$ all bit string

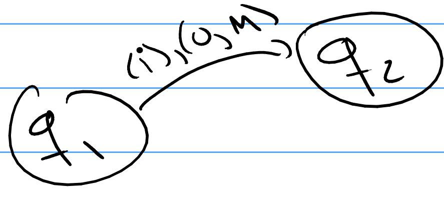
Computability

① __Turing Machines__ (sets & 5-tuples)

(q_1, i, o, M, q_2)

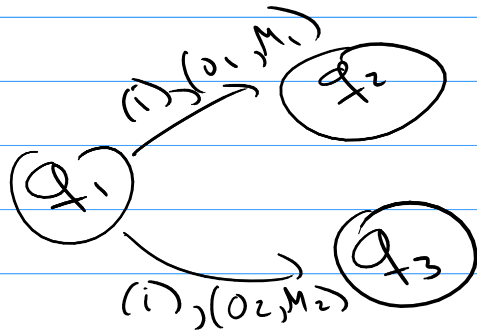
__Deterministic__

$|S| = n$



__Non-Det__

__$P(S)$__



Not more powerful (no new probs)

__but__ "faster" b/c $\log_2$ reduction in states.

② Church-Turing Thesis — Not a theorem

Any problem that can be solved with an $\boxed{\text{effective algorithm}}$, then is a turing machine that can solve it.

1, $\boxed{2}$, $\boxed{3}$, ~~4~~, $\boxed{5}$, ~~6~~, $\boxed{7}$, ~~8~~, ~~9~~, ~~10~~

$\boxed{11}$, ~~12~~, $\boxed{13}$, ~~14~~, ~~15~~, ~~16~~, $\boxed{17}$, ...

effective algorithm

$\boxed{\text{Problem}}$

have effective algorithms
have turing machine
__Def:__ solvable / decidable

do not have effective algorithms
do not have a turing machine
__Def:__ unsolvable
undecidable

Focus

problem

have turing machine ← problem → not have a turing machine

solvable	unsolvable

← all probs

ex **problem:** given a turing machine and some input to give that machine is there an effective algorithm to find if the machine and input halt or loop?

pf no effective algorithm (halting problem)

↑ proof by contradiction

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solvable vs unsolvable ?? $\{0,1,2,3,\dots\}$

Term number theory functions $f(n_1, n_2, \dots, n_k) = n_{k+1}$ n_i are non-neg ints.

ex $1+2=3$

ex $1+2+3=6$

ex consider $r = 0.12112111211112111112\dots$

$f(1)=1$ $f(3)=1$ $f(5)=2$
$f(2)=2$ $f(4)=1$ $f(6)=1$ ... $f(n) = n^{th}$ decimal

$|$ number theory functions $| =$ uncountably infinite

function

$\exists$ turing machine
$\exists$ effective algorithm
$\exists$ program to do it

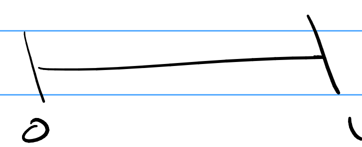
use finite sets of symbols concatenated together.

countably infinite

$\neg$ turing machine
no program, no eff. algor.

uncountably infinite

ex



$r \in \mathbb{R}$

$(\sqrt{2} - 1) = 0.$ ______

has effective algorithm $=$ computable number

$r = 0.$ ______

no effective algorith $=$ uncomputable number

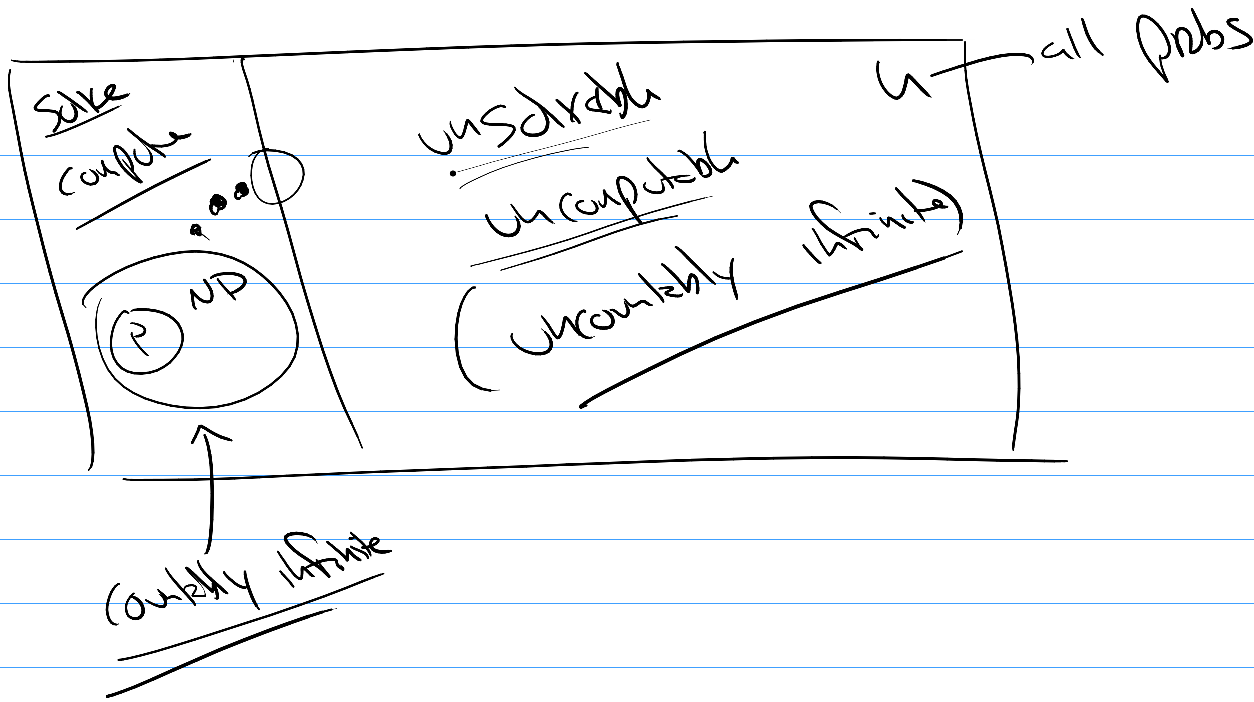
Busy Beaver Problem

$\Lambda = \{1, \#\}$ $\qquad |S| = n \qquad B(n)$

$B(2) = 4$
$B(3) = 6$
$B(4) = 13$

$B(5) \geq 4098$
$B(6) \geq 3.5 \times 10^{18267}$
$B(7) \geq 10^{10^{10^{10^{18\ \text{million}}}}}$

$\boxed{\text{all}}$ n-state Turing machines given a blank tape, $B(n)$ is max number of 1's on tape for halting machines.



P: polynomial time have a det turing machine that can solve it in polynomial time (tractable)

NP: in tractable

NP: Non-det turing machine that solves in polynomial time.