Math 322

Final Exam

4 prbs / exam 1 to 3 so 12 prbs

- WSU Finals Schedule is
  - Wed Dec 9th $\boxed{\text{1pm to 3pm}}$
  - Our final is online so .. Wed Dec 9th 12pm – 4pm
- Final is 12 problems taken from exam 1 to 3 / 2 hrs
  - Study? Solve Exam 1 to Exam 3!

Q's

Math 322 - Exam 3

Key

0) Exam Start Time:

0) MyWSUid and Name:

1) Describe the language generated by the following expressions

a) $0^*(101)1^*$

0, 1, 2, 3, ...

+5 all strings starting w/ any number of zero's contain 101 and end w/ any number of ones.

ex 0101

b) $(0|1)^*111$

+5 Any string ending with 3 ones.

ex 01101010111

2) Give a regular expression that will generate the following languages

a) Any bit string with 111 in it.

+5 $(0|1)^* 111 (0|1)^*$

b) Any bit strings that start with 01 and end with 101.

+5 $01(0|1)^*101$

Symbols | $0|1$ is $0 \cup 1$ is $0+1$

$0 \cdot 1$ is 01

ex Start with 1 or more zeros

$\boxed{0^n(0|1)^*,\ n=1,2,\ldots}$

$\boxed{00^*(0|1)^*}$ 0, 1, 2, ...

$\lambda = \varepsilon$

3) Name the grammar type (give its type number and name) and circle the productions that prevent it from being the next type.

"Strictly" not 1 so phrase structure

not according to our book though.

a) $S \to A$, $S \to B$, $S \to \lambda$, $A \to Sb$, $B \to aB$, $A \to a$, and $B \to b$

not 3 not 3

type 0, 1, 2, not 3 Context free

b) $S \to AB$, $A \to aA$, $B \to bB$, $A \to a$, and $B \to b$

not 3

type 2 not 3 Context free

c) $S \to AB$, $S \to \lambda$, $B \to aAb$, $aA \to a$, and $A \to B$

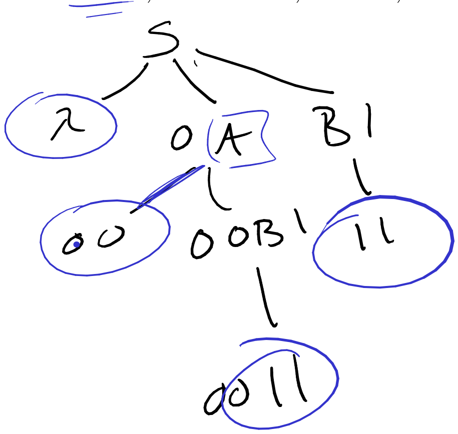
not 1

type 0 not 1 phrase structure

d) $S \to \lambda$, $S \to aA$, $A \to bB$, $B \to a$, and $A \to b$

type 3 regular grammar

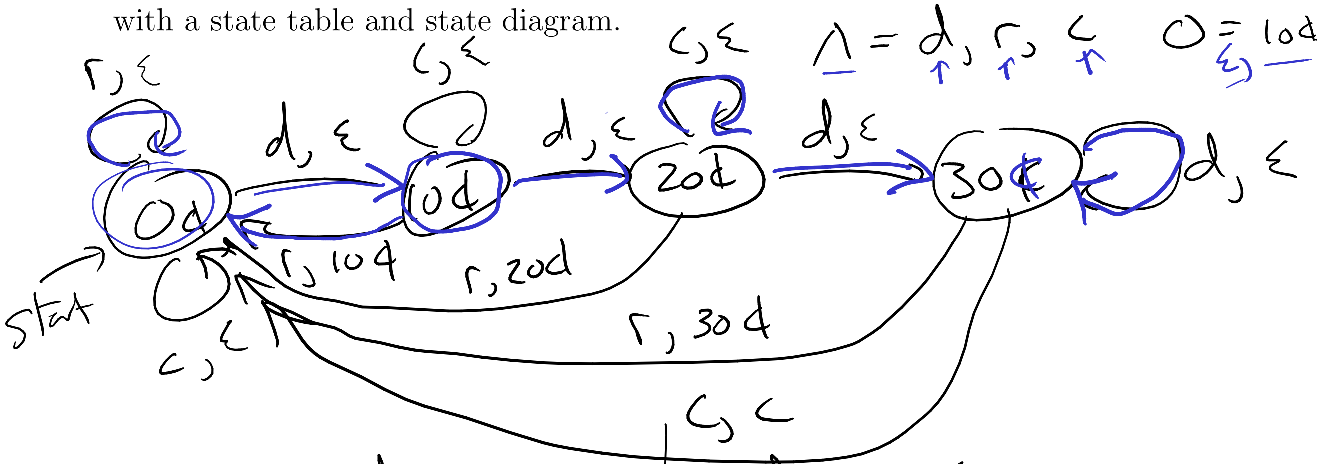
4) For the grammar with $V = \{0, 1, A, B, S\}$, $T = \{0, 1\}$, and the productions $S \to 0A$, $S \to B1$, $S \to \lambda$, $A \to 0B1$, $B \to 1$, and $A \to 0$ find L(G).



L(G) = λ | 00 | 11 | 0011

5) Construct a finite-state machine with output that models a candy machine that accepts only dimes. Candy costs 30 cents and the machine keeps the money for any amount greater than 30 cents. The customer can push buttons to receive candy or to return money. Represent the machine with a state table and state diagram.

$\Lambda = d, r, c$ $O = 10¢, 20¢, 30¢, c, \varepsilon$



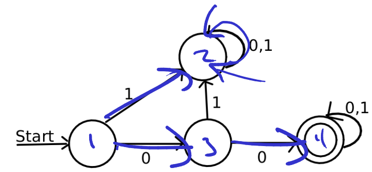
	d	r	c	d	r	c
0¢	ε	ε	ε	10¢	0¢	0¢
10¢	ε	10¢	ε	20¢	0¢	10¢
20¢	ε	20¢	ε	30¢	0¢	20¢
30¢	ε	30¢	c	30¢	0¢	0¢

outputs transitions

6) For your machine in problem 5 what is the output and the state transitions if the customer's input string is "dime,return money, dime, dime, dime, dime, return money, dime, dime, candy"?

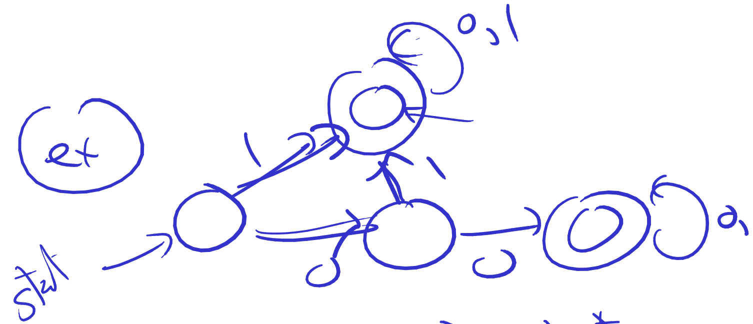
output	ε	10¢	ε	ε	ε	ε	30¢	ε	ε	ε
State	10¢	0¢	10¢	20¢	30¢	30¢	0¢	10¢	20¢	20¢

7) Determine the language recognized by a given deterministic finite-state automaton.



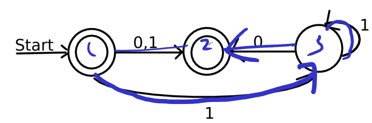
$00(0|1)^*$

Final state: S_4



$(1|01|00)(0|1)^*$

8) Determine the language recognized by a given non-deterministic finite-state automaton.



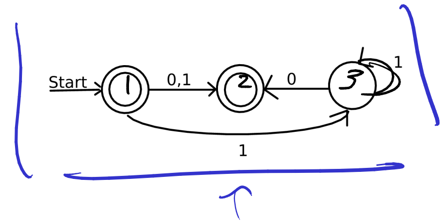
S_1 $\boxed{\varepsilon \text{ stay @ start}}$

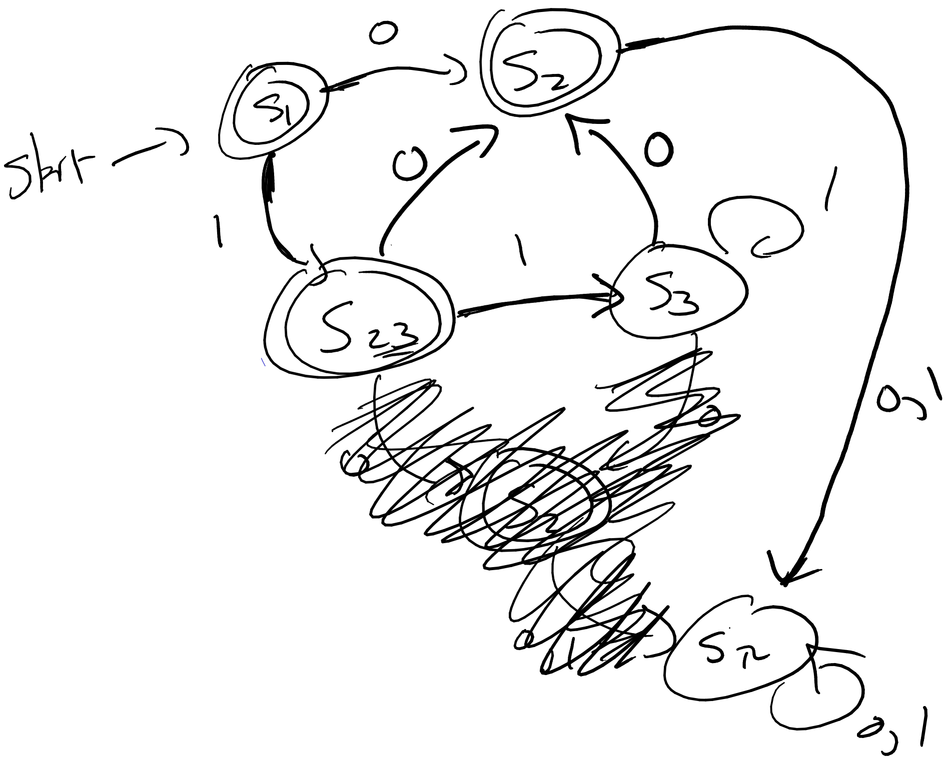
S_2 $0|1$ along top left

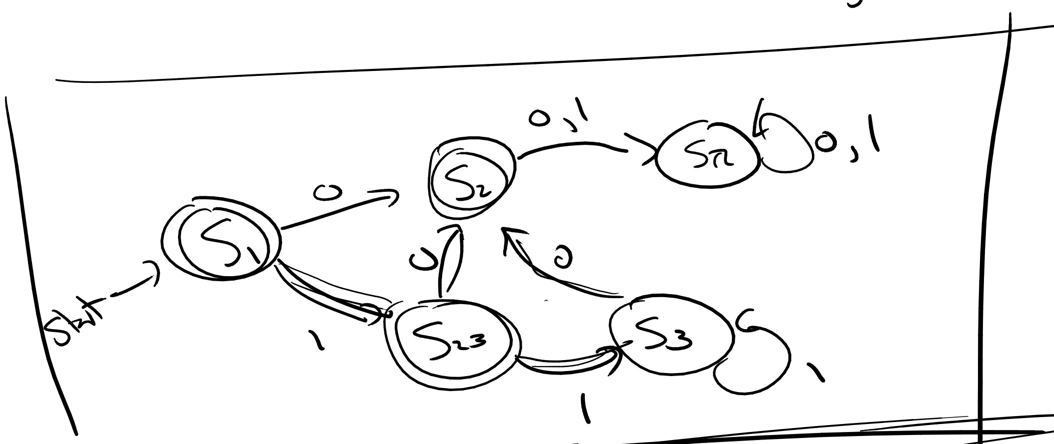
$1\,1^*0$ from right

so $\boxed{\varepsilon \,|\, 0 \,|\, 1 \,|\, 11^*0} = L(M)$

9) For the non-deterministic finite-state automaton in problem 8 construct a deterministic finite-state automaton that recognizes the same language.



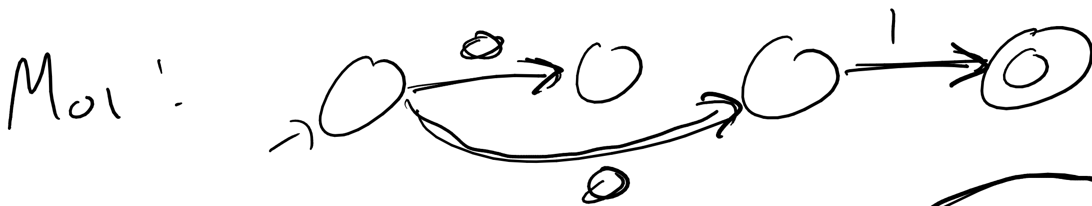


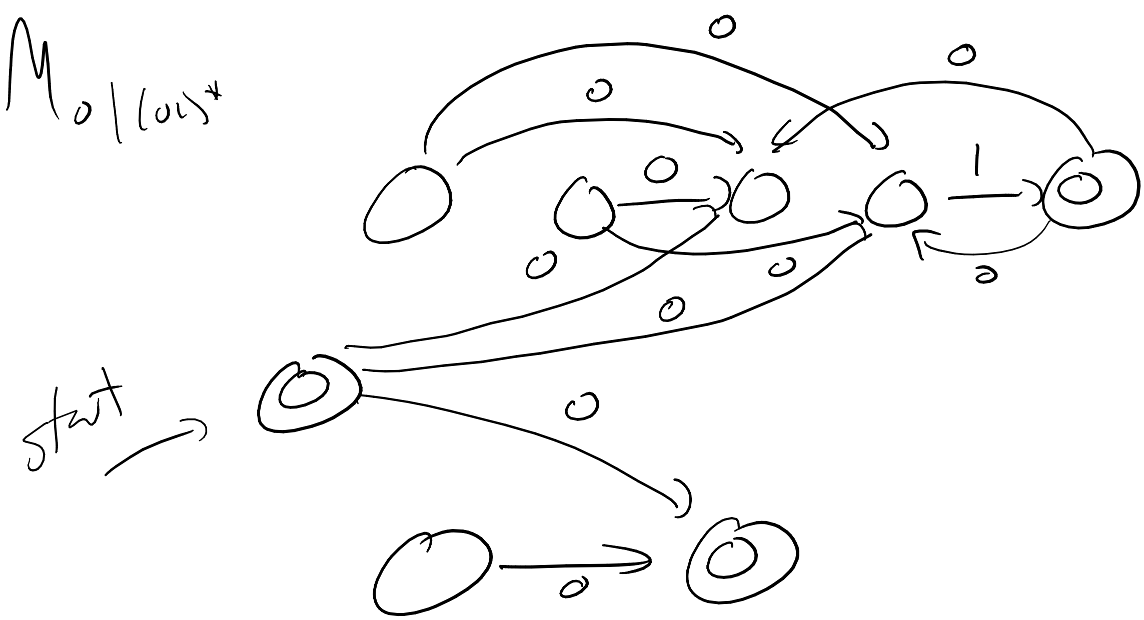


for fun $L(A) = \varepsilon \mid 0 \mid 11 \mid 10 \mid 110 \mid 11^*0$ 2, 3, 4, ...

choice $\varepsilon \mid 0 \mid 11 \mid \boxed{11^*0}$

10) Using the inductive constructions, find a non-deterministic finite-state automaton that recognizes $0 \cup (01)^*$.





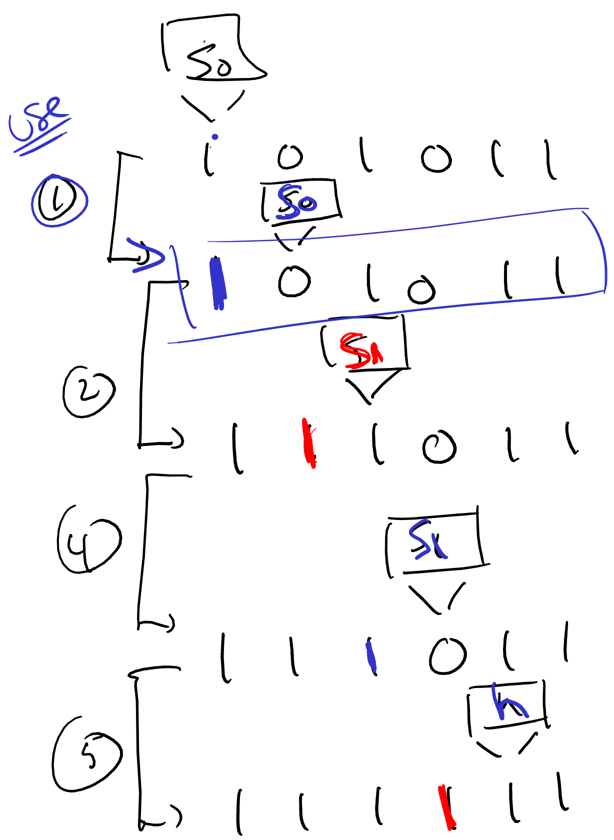
① $(s_0, 1) \to (1, R, s_0)$

11) For the Turing machine defined by the five-tuples:

$(s_0, 1, 1, R, s_0)$, $(s_0, 0, 1, R, s_1)$, $(s_0, x, 1, R, s_1)$, $(s_1, 1, 1, R, s_1)$, $(s_1, 0, 1, R, h)$, and $(s_1, x, 1, R, h)$

Run the Turning machine on the below initial tape, write each of the positions, and determine the tape when T halts. What do you think T is doing?

Inital Tape: ... #, #, #,1,0,1,0,1,1,#, #, #, ...



halts

s_0 ≡ have not seen 0

s_1 ≡ replaced 1^{st} 0 w/ 1

h ≡ replaced 2^{nd} 0 w/ 1 or replaced blank after string w/ 1.

$n = 1, 2, 3, \ldots ?$ $n = 0, 1, 2, 3, \ldots ?$ $4 \bmod 2 = 0$

12) Construct a Turing machine for $f(n) = n \bmod 2$. Run your machine on the input: 1,1,1,1,1.

0 is all blank tape
1 is 1
2 is 11
3 is 111
⋮
positives

or

0 = 1
1 = 11
2 = 111
3 = 1111
non-neg's

if this is positive ints it is 5. If non-neg ints it is 4.

assume non-neg.

$S_0 \equiv$ have not "shared" yet (or back to start of sharing)

$S_1 \equiv$ one given out S_0

$(S_0, 1) \to (\#, R, S_1)$
$(S_1, 1) \to (\#, R, S_0)$
$(S_0, \#) \to (1, R, S_1)$
$(S_1, \#) \to (1, R, h)$

4 = 1 1 1 1 1 # # #
3 = # 1 1 1 1 # # # (S_1)
2 = # # 1 1 1 # # # (S_0)
1 = # # # 1 1 # # # (S_1)
0 = # # # # 1 # # # (S_0)
$\square$ = # # # # # # # # (S_1, h)
0 = # # # # # 1 # #

$4 \bmod 2 = 0$