Math 344

- On cheating ...

"Couldn't someone share their downloaded exam with others?"

My expectations / advice

1st: you know the consequences (to you)

2nd: you know yourself

3rd: you know your classmates

4th: Real World vs In School

Self Grading

0 to 10

$\int x e^x \, dx = x e^x = -$

$u = x \qquad du = dx$

$dv = e^x dx \qquad v = e^x$

MATH 344 ... EXAM 1

0) Exam Start Time:

0) Name:

0) MyWSUid:

0) Sign below to state that you have read and understand the WSU policy on Student Academic Integrity

https://www.wichita.edu/about/policy/ch_02/ch2_17.php

0) NOTE: For problems that state "Do not simplify" I mean that you should take the derivatives but do not simplify the result.

4 7 -6

1) Show that the curve $\langle t^2, 1-3t, 1+t^3\rangle$ passes through (1, 4, 0) but not through (4, 7, -6).

x: $t^2=1 \to t=\pm 1$

y: $1-3t=4 \to t=-1$

z: $1+t^3=0 \to t=-1$

so @ $t=-1$ pt is $(1,4,0)$

x: $t^2=4 \to t=\pm 2$

y: $1-3t=7 \to t=-2$

z: $1+t^3=-6 \to t=(-7)^{1/3}$

no soln

no solution so it doesn't pass through $(4,7,-6)$

2) Find the limits ...

a)

$$\lim_{t\to 2}\left\langle t^2-2, \frac{t^2-4}{t-2}, \sin(t)\right\rangle = \boxed{\langle 2, 4, \sin(2)\rangle}$$

$$\lim_{t\to 2} t^2-2 = 2^2-2 = 2$$

$$\lim_{t\to 2} \frac{t^2-4}{t-2} = \lim_{t\to 2} (t+2) = 4$$

$$\lim_{t\to 2} \sin(t) = \sin(2)$$

b)

$$\lim_{t\to\infty}\left\langle \frac{t^2+t}{3t^2-1}, t\sin\left(\frac{1}{t}\right)\right\rangle = \boxed{\left\langle \frac{1}{3}, 1\right\rangle}$$

$$\lim_{t\to\infty} \frac{t^2+t}{3t^2-1} = \lim_{t\to\infty} \frac{1+1/t}{3-1/t^2} = \frac{1+0}{3-0} = \frac{1}{3}$$

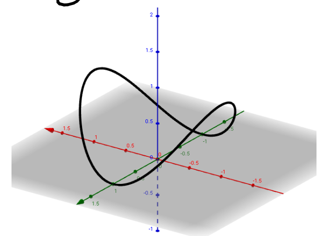
$$\lim_{t\to\infty} t\sin\left(\frac{1}{t}\right) = \lim_{t\to\infty} \frac{\sin\left(\frac{1}{t}\right)}{1/t} = \lim_{x\to 0^+} \frac{\sin x}{x} = 1$$

$$\lim_{t\to\infty} x = \frac{1}{t}$$

$$x \to 0^+$$

3) Describe the space curve $\boldsymbol{r}(t) = \langle \sin(t), \cos(t), \sin^2(t) \rangle$ and give reasons for your description.

$\langle x(t), y(t) \rangle = \langle \sin(t), \cos(t) \rangle$ is a circle around origin of xy plane.  $z(t) = \sin^2(t)$ goes between 0 and 1 ~~So~~ it looks like the edge of a pringles chip.



4) If $r(t) = \langle t^3\sqrt{1+t^2}, \sin^{2/3}(1-2t), (t+t^2)/(1-t) \rangle$ find $\boldsymbol{r}'(t)$. DO NOT simplify your answers.

$$x'(t) = \boxed{3t^2\sqrt{1+t^2} + t^3 \frac{1}{2\sqrt{1+t^2}} \cdot 2t}$$ ← note a $\sqrt{\ }$

$$y'(t) = \boxed{\frac{2}{3}\sin^{-1/3}(1-2t)\cos(1-2t)\,(-2)}$$ ← note a trig

$$z'(t) = \boxed{\frac{(1+2t)(1-t) - (t+t^2)(-1)}{(1-t)^2}}$$ ← note a factoring

$$r' = \langle x', y', z' \rangle$$

5) Evaluate the integral ...

$$\int \left\langle te^{3t+1}, \frac{\cos(t)}{\sin^2(t)-1} \right\rangle dt$$

$$\int t e^{3t+1}\,dt = \frac{1}{3} t e^{3t+1} - \frac{1}{3}\int e^{3t+1}\,dt = \boxed{\frac{1}{3} t e^{3t+1} - \frac{1}{9} e^{3t+1} + C_1}$$

$u = t \rightarrow du = dt$

$dv = e^{3t+1}\,dt \rightarrow v = \frac{1}{3} e^{3t+1}$

$$\int \frac{\cos(t)}{\sin^2(t)-1}\,dt = \text{See next page}$$

$$\boxed{\left\langle \frac{1}{3} t e^{3t+1} - \frac{1}{9} t e^{3t+1}, -\ln|\sec t + \tan t| \right\rangle + C}$$

6) Evaluate the definite integral ...

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\int_0^1 \left\langle \frac{1+t}{1+t^2}, 3t - t^2 + t^3 \right\rangle dt = \boxed{\left\langle \frac{\pi}{4} + \frac{1}{2}\ln(2), \frac{17}{12} \right\rangle}$$

$$\int_0^1 \boxed{\frac{1+t}{1+t^2}}\,dt = \int_0^1 \frac{1}{1+t^2}\,dt + \int_0^1 \frac{t}{1+t^2}\,dt$$

$u = 1+t^2$

$du = 2t\,dt$

$$= \left[\arctan(1) - \arctan(0)\right] + \frac{1}{2}\int_1^2 \frac{1}{u}\,du$$

$$= \boxed{\frac{\pi}{4} + \frac{1}{2}\ln(2)} =$$

$$\int_0^1 3t - t^2 + t^3\,dt = \frac{3}{2} - \frac{1}{3} + \frac{1}{4} = \frac{18-4+3}{12} = \boxed{\frac{17}{12}}$$

$$\int \frac{\cos(t)}{\sin^2(t)-1}\,dt = \int \frac{1}{u^2-1}\,du \overset{B}{=} \frac{1}{2}\int \left(\frac{-1}{u+1} + \frac{1}{u-1}\right) du$$

$$= \frac{1}{2}\left[-\ln|u+1| + \ln|u-1|\right] + C_2$$

$$= \frac{1}{2}\ln\left|\frac{u-1}{u+1}\right| + C_2$$

$$= \boxed{\frac{1}{2}\ln\left|\frac{\sin(t)-1}{\sin(t)+1}\right| + C_2}$$

$u = \sin(t)$

$du = \cos(t)\,dt$

$$\frac{A}{u+1} + \frac{B}{u-1}$$

$$\frac{A(u-1)+B(u+1)}{u^2-1}$$

$A + B = 0$

$-A + B = 1$

$2B = 1 \quad B = 1/2 \quad A = -1/2$

$$\int \frac{\cos(t)}{\sin^2(t)-1}\,dt = -\int \frac{\cos(t)}{\cos^2(t)}\,dt = -\int \sec(t)\,dt$$

$$= -\ln|\sec t + \tan t| + C_2$$

mult by $\dfrac{\sec(t)+\tan(t)}{\sec(t)+\tan(t)}$

CAS Maxima, geogebra, sympy

7) Setup the definite integral to find the length of the curve, but DO NOT integrate it: $\boldsymbol{r}(t) = \langle t^2, 9t, 4t^{3/2}\rangle$, $1 \leqslant t \leqslant 4$.

$r' = \langle 2t, 9, 6t^{1/2} \rangle$

$$\boxed{s = \int_1^4 \sqrt{4t^2 + 81 + 36t}\, dt}$$

$$s = \int_a^b |r'|\, dt$$

8) Reparametrize the curve $\boldsymbol{r}(t) = \langle 3t, \cos(t), \sin(t)\rangle$ with respect to arc length measured from t=0 and increasing.

$r' = \langle 3, -\sin(t), \cos(t) \rangle$

$$s(t) = \int_0^t \sqrt{9 + \sin^2 u + \cos^2 u}\, du$$

$$= \sqrt{10} \int_0^t du = \sqrt{10}\, t$$

$$\boxed{s = \sqrt{10}\, t}$$

$$\boxed{t = \frac{1}{\sqrt{10}} s}$$

So $r(t) = \langle 3t, \cos(t), \sin(t) \rangle$ is also

$$\boxed{r(s) = \left\langle \frac{3}{\sqrt{10}} s, \cos\left(\frac{1}{\sqrt{10}} s\right), \sin\left(\frac{1}{\sqrt{10}} s\right) \right\rangle}$$

$$\kappa = \frac{|T'|}{|r'|}$$

9) Find the curvature for the circle of radius 10 $\boldsymbol{r}(t) = \langle 10\cos(t), 10\sin(t)\rangle$.

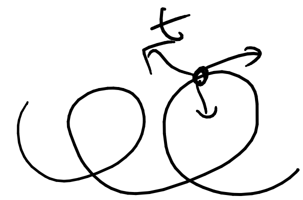
$$r' = \langle -10\sin t, 10\cos t\rangle \qquad |r'| = \sqrt{100\sin^2 t + 100\cos^2 t} = 10$$

$$T = \langle -\sin t, \cos t\rangle$$

$$T' = \langle -\cos t, -\sin t\rangle \qquad |T'| = \sqrt{\cos^2 t + \sin^2 t} = \sqrt{1} = 1$$

$$\boxed{\kappa(t) = \frac{1}{10}}$$

$$\frac{|T'|}{|r'|}$$



10) For the curve given in problem 8 find $\boldsymbol{T}$,$\boldsymbol{N}$, and $\boldsymbol{B}$.

$$r(t) = \langle 3t, \cos t, \sin t\rangle$$

$$r' = \langle 3, -\sin t, \cos t\rangle \qquad |r'| = \sqrt{10} \quad (\text{See } \#8)$$

$$\boxed{T = \left\langle \frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}}\sin t, \frac{1}{\sqrt{10}}\cos t\right\rangle} = \boxed{\frac{1}{\sqrt{10}}\langle 3, -\sin t, \cos t\rangle}$$

$$N = \frac{\left\langle 0, -\frac{1}{\sqrt{10}}\cos t, -\frac{1}{\sqrt{10}}\sin t\right\rangle}{\sqrt{\frac{1}{10}}} = \boxed{\langle 0, -\cos t, -\sin t\rangle}$$

$$B = \begin{vmatrix} i & j & k \\ \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}}\sin t & \frac{1}{\sqrt{10}}\cos t \\ 0 & -\cos t & -\sin t \end{vmatrix} = \boxed{\left\langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\sin t, -\frac{3}{\sqrt{10}}\cos t\right\rangle}$$

$$= \boxed{\frac{1}{\sqrt{10}}\langle 1, 3\sin t, -3\cos t\rangle}$$

$\cancel{t=0}$

T, N, B

11) For the curve $\boldsymbol{r}(t) = \langle t^2, 1-3t, 1+t^3\rangle$ find the position, velocity, and acceleration at t = -1.

$\mathbf{v}(t) = \langle 2t, -3, 3t^2\rangle$

$\mathbf{a}(t) = \langle 2, 0, 6t\rangle$

$$\boxed{\begin{aligned} \mathbf{r}(-1) &= \langle 1, 4, 0\rangle \\ \mathbf{v}(-1) &= \langle -2, -3, 3\rangle \\ \mathbf{a}(-1) &= \langle 2, 0, -6\rangle \end{aligned}}$$

12) For a projectile fired at a speed of v_0 with an angle of elevation α its acceleration is

$$\boldsymbol{a}(t) = \langle 0, -g\rangle$$

and its initial velocity vector is $\langle v_0\cos(\alpha), v_0\sin(\alpha)\rangle$. If it is fired from the ground what is the time to maximum height and what is the maximum height?

$\mathbf{v}(t) = \int \mathbf{a}(t)\,dt = \langle C_x, -gt + C_y\rangle$

$\mathbf{v}(t) = \langle v_0\cos(\alpha), -gt + v_0\sin(\alpha)\rangle$ + ~~C~~ 0

no 2

$\mathbf{r}(t) = \langle v_0\cos(\alpha)t + 0, -\frac{1}{2}gt^2 + v_0\sin(\alpha)t + \cancel{C_y}\ 0\rangle$

Max height is @ $-gt + v_0\sin(\alpha) = 0 \to \boxed{t = \frac{v_0\sin(\alpha)}{g}}$

So Max height

$\uparrow$

y of $\mathbf{r} = \langle x, y\rangle$

$$\left|\ -\frac{1}{2}g\left(\frac{v_0\sin(\alpha)}{g}\right)^2 + v_0\sin(\alpha)\left(\frac{v_0\sin(\alpha)}{g}\right)\right.$$

My typical copy errors tend to take from similar expressions.

I had a 2 here before.

0) What is the time you ended working on the exam and started scanning it?