# Math 344 (Calc 3)

Q's No webassign HW for 14.1

$x_{n+1} = f(x_1, x_2, \dots, x_n)$

Domain: $\mathbb{R}^n$ ($n$ Dimensional, $n$-D)

$(x_1, x_2, \dots, x_n)$ function $\rightarrow$ $(\underbrace{x_1, x_2, \dots, x_n}_{\text{domain}}, \underbrace{x_{n+1}}_{\text{codomain}})$

Codomain: $\mathbb{R}$ $x_n$

ex

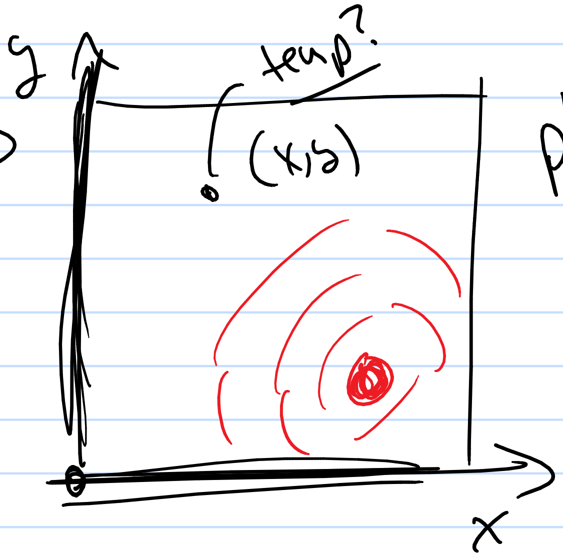



plate of metal

$T(x,y)$ = temp @ $(x,y)$

in 3D $(\underbrace{x, y}_{\text{domain}}, \underbrace{T}_{\text{codomain}})$

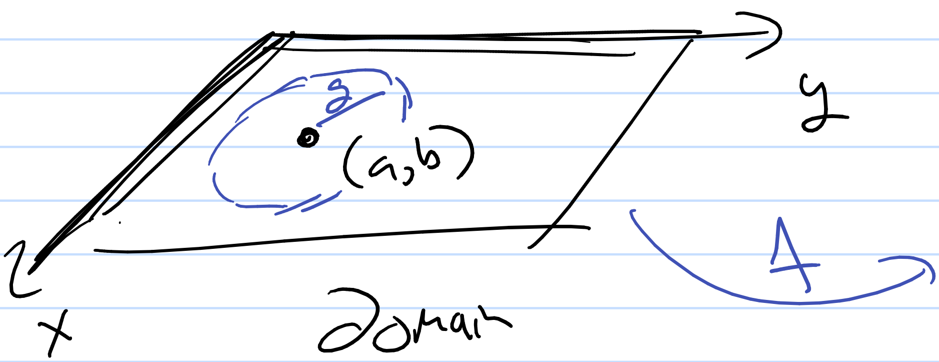
calculus on $x_{n+1} = f(x_1, x_2, \dots, x_n)$

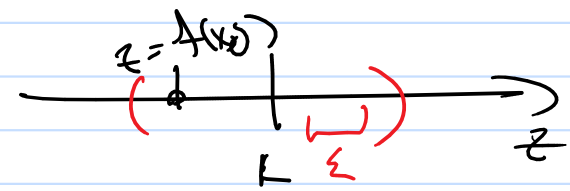
#1 limits

(ex) $z = f(x,y)$

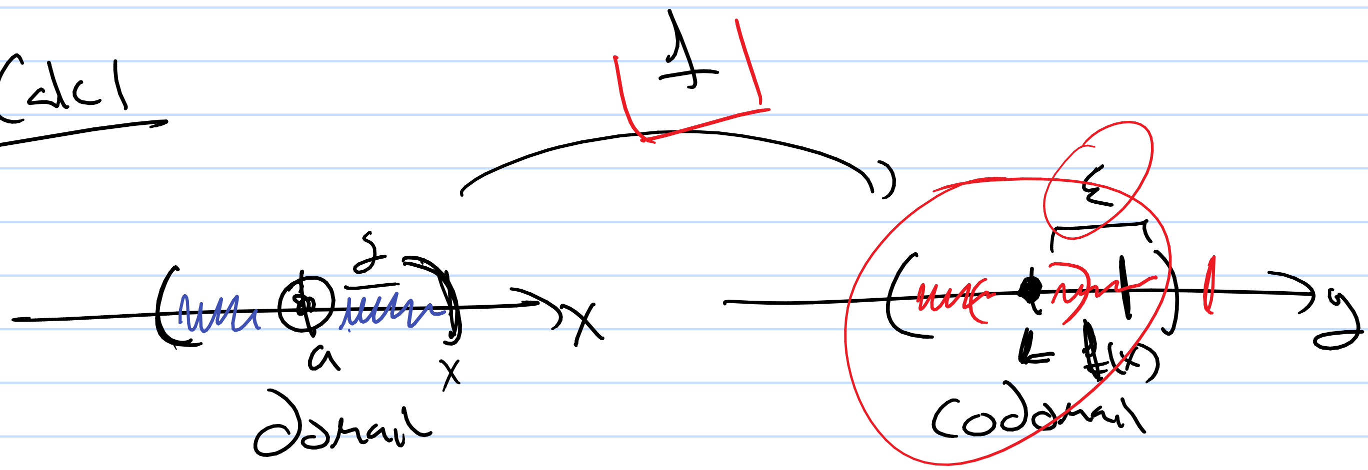
as if $0 < \sqrt{(x-a)^2+(y-b)^2} < \delta$, then

$|\underbrace{f(x,y)} - L| < \varepsilon$




(ex) Calc I




$$\lim_{x\to a} f(x) = L.$$

if $\underline{0<|x-a|<\delta}$, then $|f(x)-L|<\varepsilon$

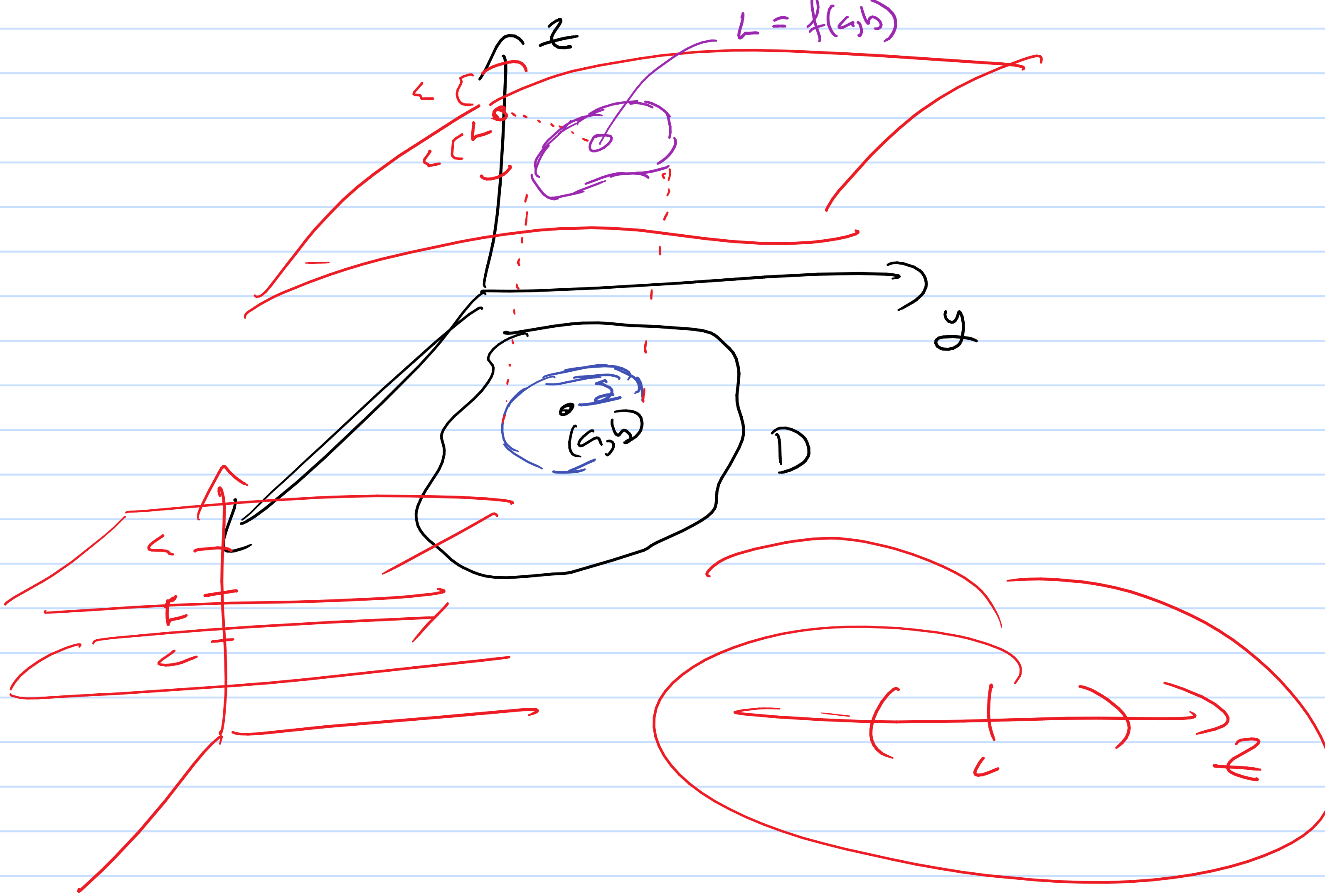
$\sqrt{(x-a)^2}$

So for $z = f(x,y)$

**Def:** $\lim_{(x,y)\to(a,b)} f(x,y) = L$ D is a region around $(a,b)$

$(x,y) \in D$

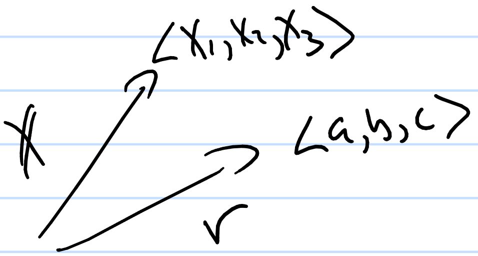
if $0 < \sqrt{(x-a)^2+(y-b)^2} < \delta$, then $|f(x,y) - L| < \varepsilon$



What about going up in dimension?

$$\lim_{(x_1,x_2,x_3)\to(a,b,c)} f(x_1,x_2,x_3) = L$$

$$0 < \sqrt{(x_1-a)^2+(x_2-b)^2+(x_3-c)^2} < \delta \quad \text{then} \quad |f(\underbrace{x_1,x_2,x_3}_{\mathbf{x}})-L|<\varepsilon$$




$$|\mathbf{x}-\mathbf{r}|$$

For any dimension

$$\boxed{\text{if } 0 < |\mathbf{x}-\mathbf{r}| < \delta, \text{ then } |f(\mathbf{x})-L| < \varepsilon}$$

prove?

$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

$$0 < \sqrt{(x-a)^2+(y-b)^2} < \delta \quad \text{then} \quad |f(x,y)-L|<\varepsilon$$

**Note:** $\frac{x^2}{x^2+y^2} < \frac{x^2}{x^2} = 1$

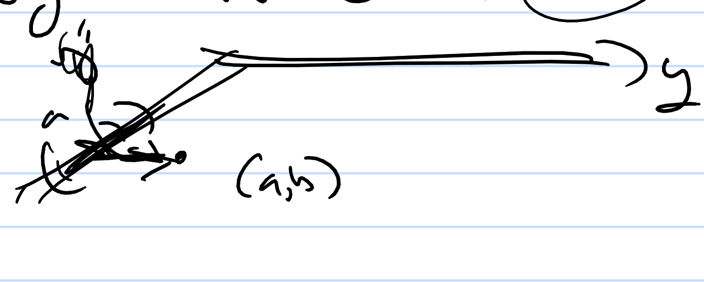
$$\frac{x^2}{x^2+y^3}\, y < y$$

---

to show a limit does not exist

show ① $(x,y) \to (a,b)$ along a specific curve $C_1$ that $f(x,y) \to L_1$

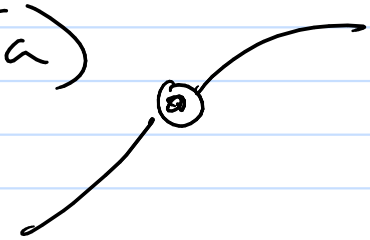
② $(x,y) \to (a,b)$ along another curve $C_2$ that $f(x,y) \to L_2$




$\boxed{\text{and}}$ $L_1 \neq L_2$ limit does not exist

## Continuous functions:

**Calc I** $\lim\limits_{x\to a} f(x) = f(a)$



**Now:** $\lim\limits_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

if $f(x,y)$ is polynomial like.
it is cont.

if $f(x,y)$ is rational it is cont where denom $\neq 0$.

$$\lim_{(x,y)\to(1,2)} x^2+xy-y^2 = 1^2+1\cdot 2-2^2$$

$$\lim_{(x,y)\to(1,2)} \frac{x^2}{x^2+y^2} = \frac{1^2}{1^2+2^2}$$

change?, (derivatives)