Math 3414

Partials

who is variable? (explicit vs implicit)

who is constant?

ex $\boxed{z} = \boxed{3xy^2 + \sin(x^3 + y^2)}$

$z = f(x,y)$

$f_x = \boxed{3y^2 + 3x^2 \cos(x^3+y^2)}$ $\boxed{f_y = 6xy + 2y\cos(x^3+y^2)}$

prod rule

$f_{xx} = \boxed{(6x)\cos(x^3+y^2) - (3x^2)(3x^2)\sin(x^3+y^2)}$

$f_{xy} = 6y - 3x^2(2y)\sin(x^3+y^2)$

$f_{yx} = 6y - 2y(3x^2)\sin(x^3+y^2)$

Implicit functions? known $\boxed{z = f(x,y)}$

$\frac{\partial z}{\partial x}$? $\frac{\partial z}{\partial y}$? $zxy = x^2 + y^2 + z^2$

Calc 1 $\quad y^3 \quad$ but $\quad y = f(x)$

$$\frac{d}{dx}\left[y^3\right] \qquad\qquad \frac{d}{dx}\left[(\sin x)^3\right]$$

$$= 3y^2 \frac{dy}{dx} \qquad\qquad = 3(\sin x)^2 (\cos x)$$

So $\quad zxy = x^2 + y^2 + z^2$

$\partial/\partial x \qquad \frac{\partial}{\partial x}\left[(z)(xy)\right] = \frac{\partial}{\partial x}\left[x^2 + y^2 + z^2\right]$

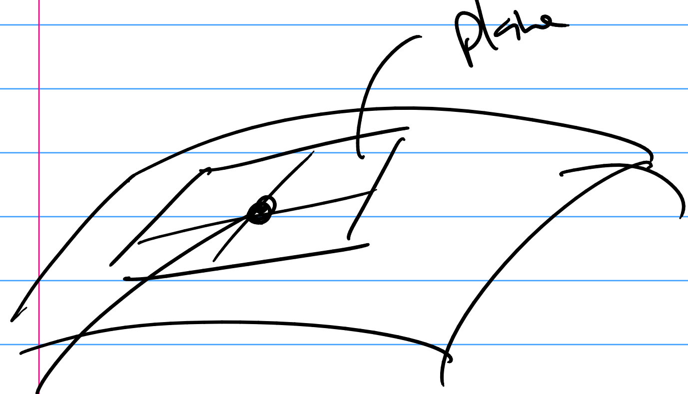
$$1\frac{\partial z}{\partial x}(xy) + zy = 2x + 2z\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x}(xy - 2z) = 2x - zy$$

$$\frac{\partial z}{\partial x} = \frac{2x - zy}{xy - 2z}$$

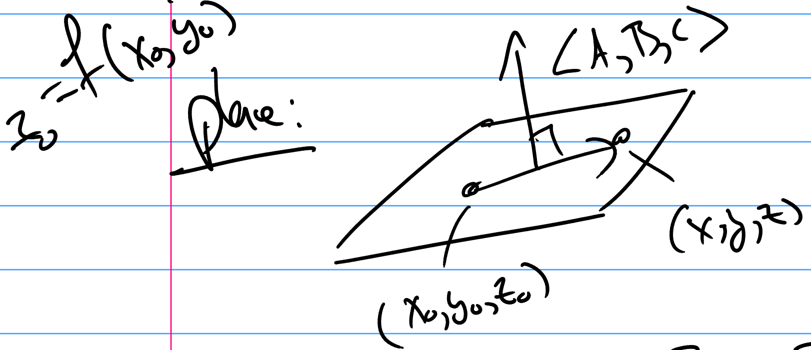
Applications:

- tangent planes
- Linear Approximations



plane:

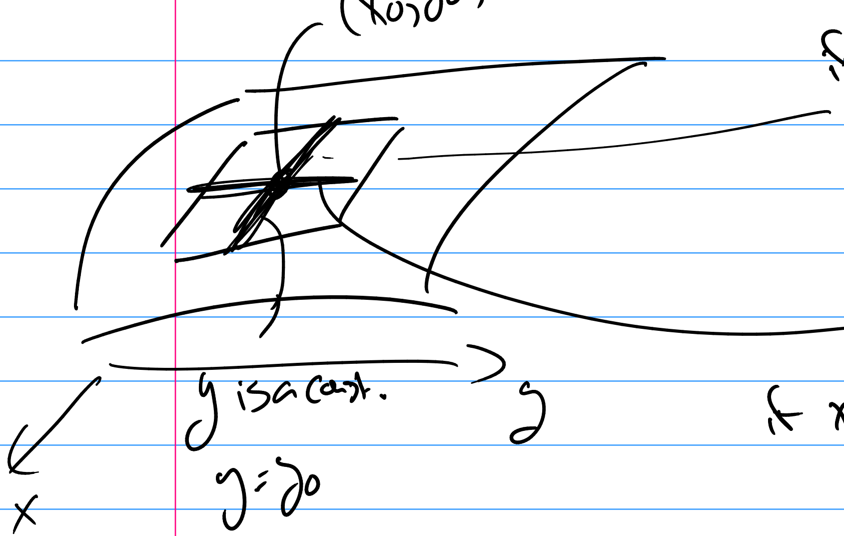
$$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$$



$$z = z_0 + \frac{A}{c}(x-x_0) + \frac{B}{c}(y-y_0)$$

plane:

$$z = f(x_0, y_0) + a(x-x_0) + b(y-y_0)$$



if $y = y_0$

$$z = f(x_0, y_0) + a(x-x_0)$$

slope $f_x(x_0, y_0)$

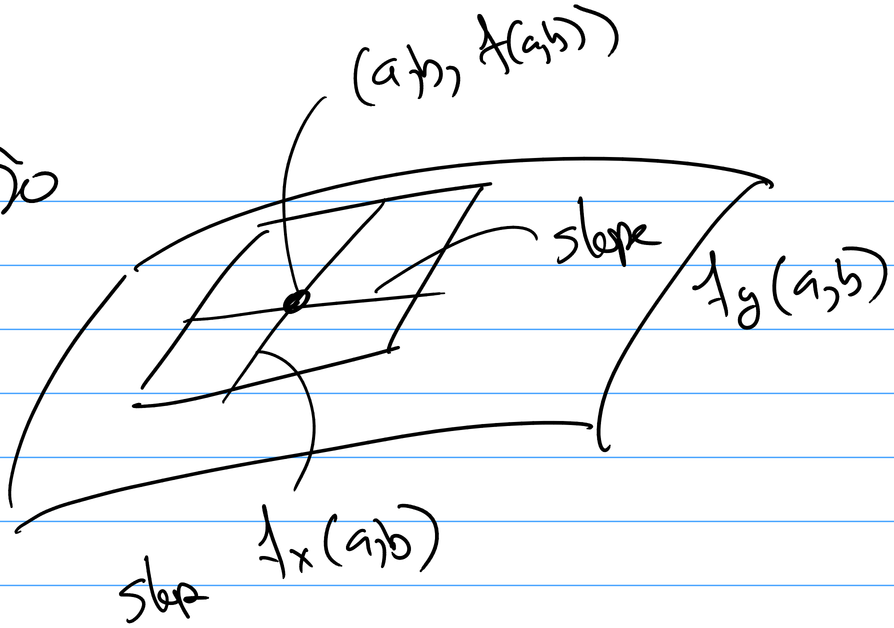
similarly

if $x = x_0$

$$z = f(x_0, y_0) + b(y-y_0)$$

slope $f_y(x_0, y_0)$

So



eqn of tangent plane @ (a,b)

$$\boxed{z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)}$$

when would tangent plane exist?

Def. Δz is difference between surface and approx. @ (a,b)

$$\Delta z = f_x(a,b)\,\Delta x + f_y(a,b)\,\Delta y + \varepsilon_1 \Delta x + \varepsilon_2 \Delta y$$

so $z = f(x,y)$ is differentiable if

$\varepsilon_1 \to 0$, $\varepsilon_2 \to 0$ as $\Delta x \to 0$ $\Delta y \to 0$

$\boxed{\text{th}^{\text{m}}}$ if f_x, f_y exist near (a,b) and continuous @ (a,b)

f is differentiable @ (a,b)

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so
$$f(x,y) \overset{\text{near } (a,b)}{\approx} f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$