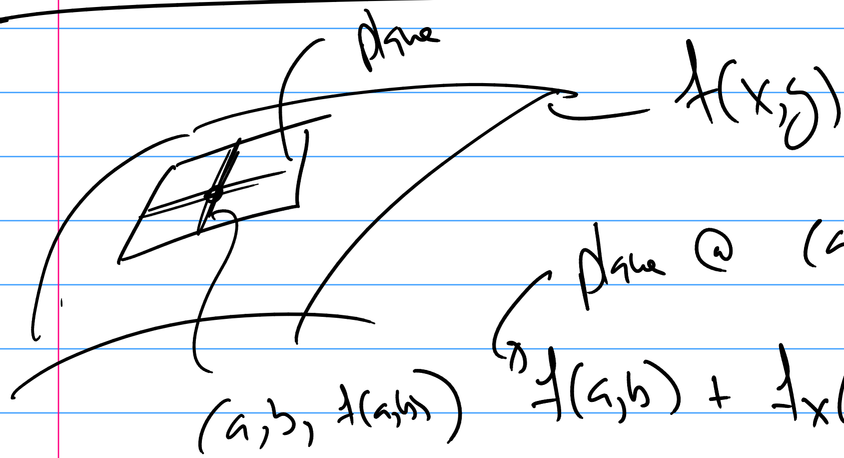
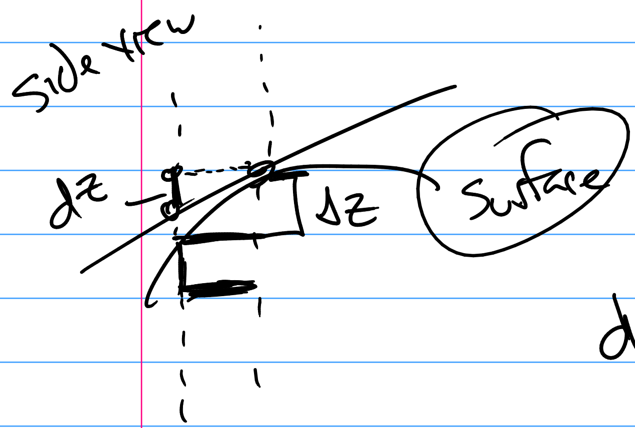
Math 3514



plane @ (a,b)

$$f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$\underline{f(x,y)} \approx \underbrace{f(a,b) + \underbrace{f_x(a,b)(x-a) + f_y(a,b)(y-b)}}_{\text{linear approximation}}$$



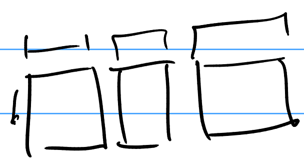
$$\Delta z \approx dz$$

$$dz = f_x\,dx + f_y\,dy$$

total differential

error prop.

measure x $\pm$ error in x $= dx$



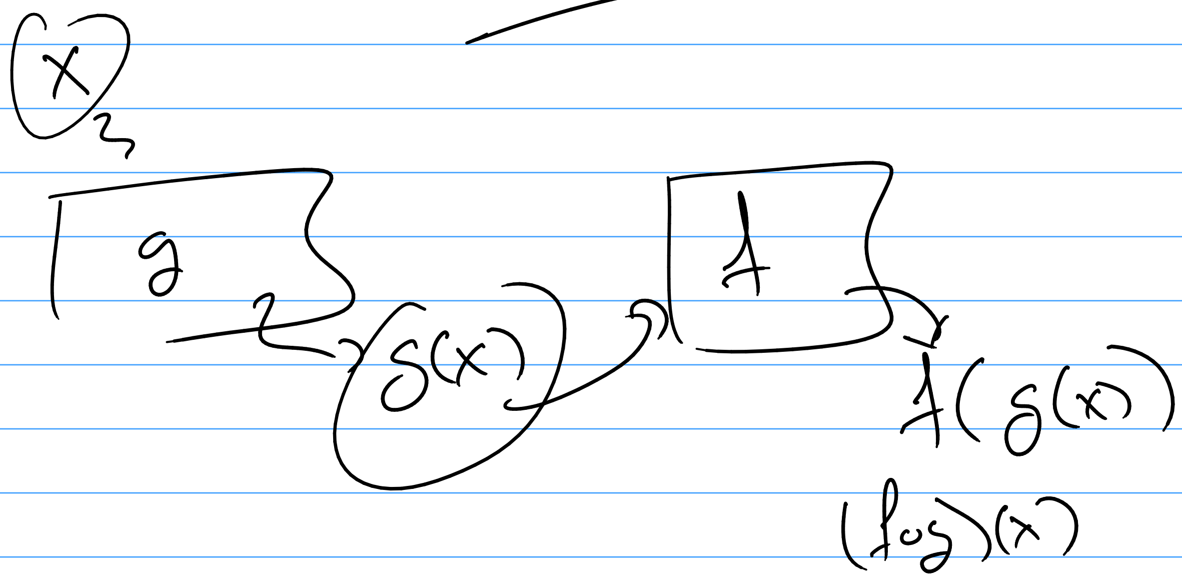
measure y $\pm$ error in y $= dy$

error in $z = f(x,y)$ $\sim$ $\Delta z \approx dz$

14.5 Chain Rule

Problem of composition

Calc 1

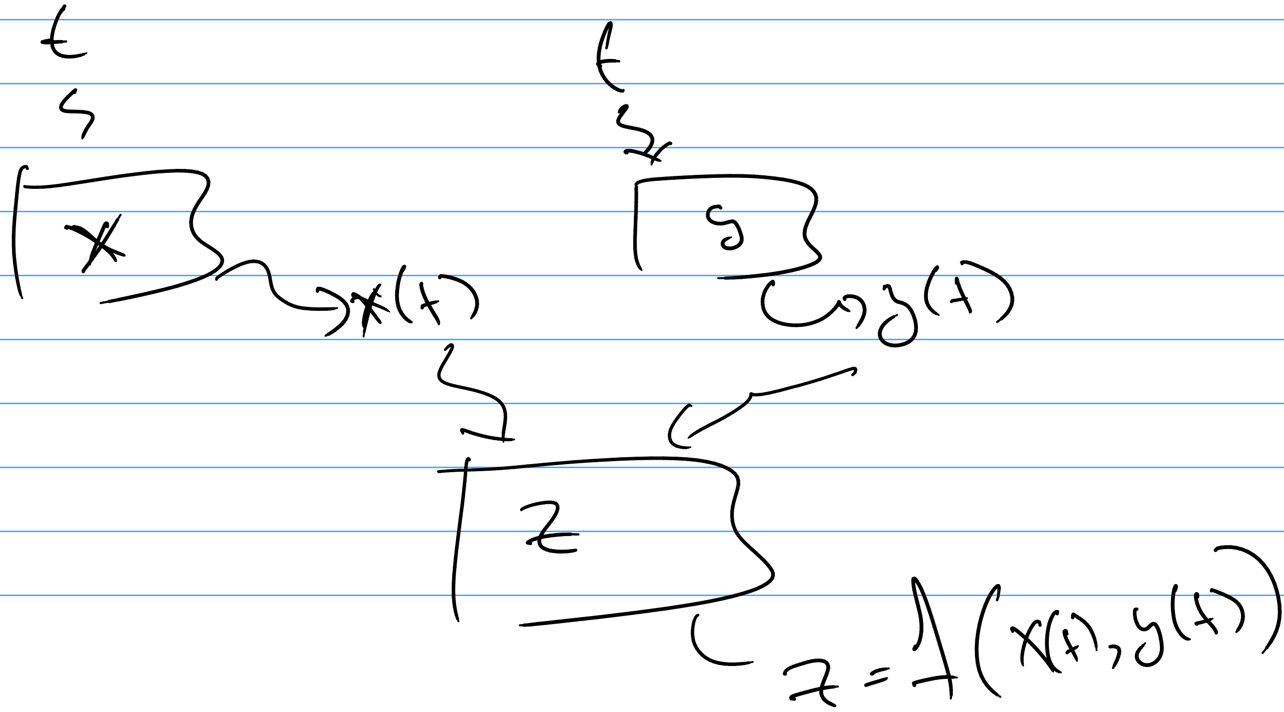


$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

Calc 3?

$z = f(x,y)$

$x = x(t) \qquad y = y(t)$



Ex $x = t^2 + 1$ $y = \sqrt{t-3}$

$z = xy + y^2$

$$\boxed{z = (t^2+1)\sqrt{t-3} + t - 3}$$

$z = z(t)$ $\frac{dz}{dt}$

Chain Rule for $z = f(x,y)$ $x = x(t)$ $y = y(t)$

$$\frac{dz}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt}$$

Ex $x = t^2 + 1$ $x' = 2t$ $y = \sqrt{t-3}$ $y' = \frac{1}{2\sqrt{t-3}}$

$$\boxed{z = xy + y^2}$$ $f_x = y$, $f_y = x + 2y$

$$\frac{dz}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt}$$

$$\boxed{\frac{dz}{dt} = y(2t) + (x+2y)\frac{1}{2\sqrt{t-3}}}$$

$$x_{n+1} = f(x_1, x_2, x_3, \dots, x_n)$$

<u>and</u> $x_1(t), \; x_2(t), \; \dots \;, x_n(t)$

$$\frac{d\,x_{n+1}}{dt} = f_{x_1}\frac{dx_1}{dt} + f_{x_2}\frac{dx_2}{dt} + \dots + f_{x_n}\frac{dx_n}{dt}$$

$z = f(x,y)$ $\qquad$ $x = x(t,s)$

$\qquad\qquad\qquad\quad\;\; y = x(t,s)$

$$z = f(x,y) = f\big(x(t,s), y(t,s)\big)$$

so $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$

<u>chain rule</u>

$$\frac{\partial z}{\partial s} = f_x\frac{\partial x}{\partial s} + f_y\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = f_x\frac{\partial x}{\partial t} + f_y\frac{\partial y}{\partial t}$$

So $z = f(x_1, x_2, x_3, \dots, x_n)$ <u>and</u> $x_i = x_i(t_1, t_2, \dots, t_m)$

$$z_{t_1} = f_{x_1}\frac{\partial x_1}{\partial t_1} + f_{x_2}\frac{\partial x_2}{\partial t_1} + f_{x_3}\frac{\partial x_3}{\partial t_1} + \dots$$

$$\vdots$$

$$z_{t_m} = f_{x_1}\frac{\partial x_1}{\partial t_m} + \dots$$