Math 344

Note: Finding extrema

f4.7 Solve: $\begin{cases} f_x = 0 \\ f_y = 0 \end{cases}$

system of eqn's

ex

$f(x,y) = 2 - x^4 + 2x^2 - y^2$

critical locations: Solve $\begin{cases} f_x = 0 \\ f_y = 0 \end{cases}$

$\begin{cases} -4x^3 + 4x = 0 \rightarrow -4x(x^2-1) = 0 \\ -2y = 0 \rightarrow y = 0 \end{cases}$

So $x = \pm 1, 0$ $\quad y = 0$

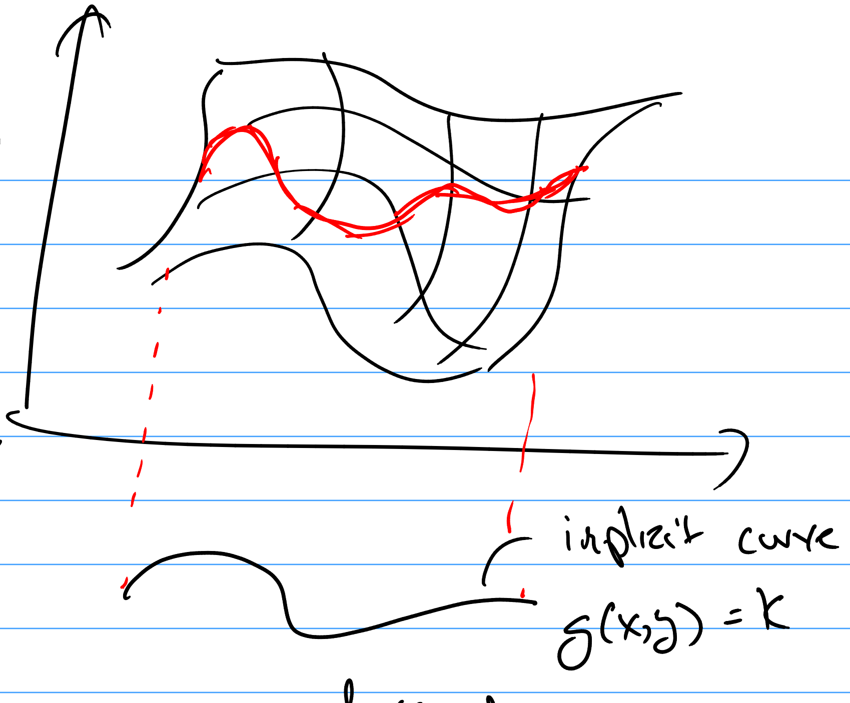
locations: $(-1,0)$, $(1,0)$, $(0,0)$

possible places of extrema

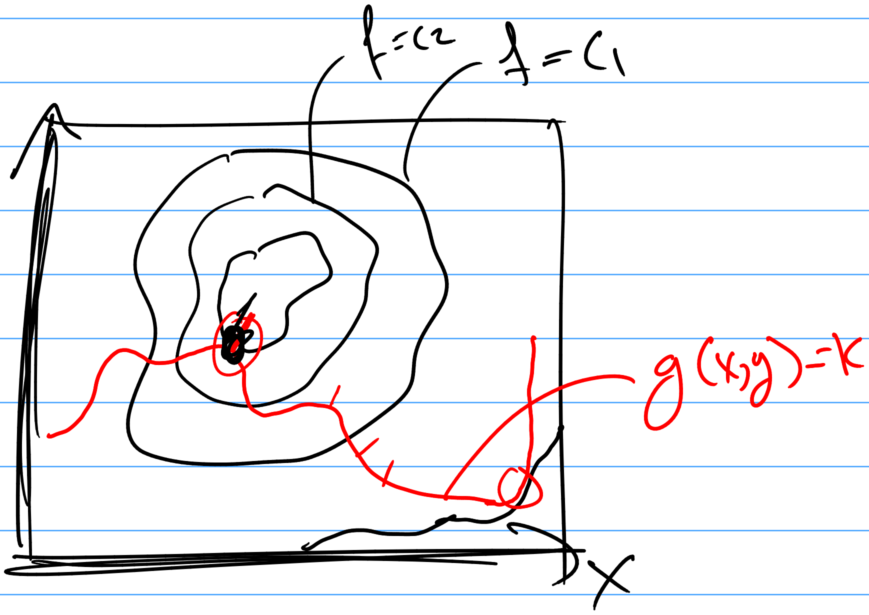
to check use $f_{xx} f_{yy} - (f_{xy})^2$ @ each location

$\boxed{14.8}$ Max/Min of $z = f(x,y)$

under constraint of $g(x,y) = K$



Visualize with level curves



We see that max/min can happen when ∇f and ∇g are $\boxed{\text{parallel}}$

same direction but different lengths

So $\nabla f = \underbrace{\lambda}_{\text{Lagrange Multiplier}} \nabla g$ (λ: scalar)

Method of Lagrange Multipliers

$f(x,y)$ or $f(x,y,z)$

Max/Min of $f(x,y)$ constrained by $g(x,y)=K$

Max/Min of $f(x,y,z)$ constrained by $g(x,y,z)=K$

① Solve:
$$\begin{cases} \nabla f = \lambda \nabla g \\ g = K \end{cases}$$

$f(x,y)$, $g(x,y)=K$

$$\langle f_x, f_y \rangle = \lambda \langle g_x, g_y \rangle$$

$$\Leftrightarrow \begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ g = K \end{cases}$$

Solve for x, y, λ

$f(x,y,z)$

$$\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ f_z = \lambda g_z \\ g = K \end{cases}$$

Solve x, y, z, λ

② find all locations ~~X~~

then evaluate f (solutions found)

largest = max

smallest = min

Ex $f(x,y,z) = \boxed{e^{xyz}}$ constant $\underbrace{2x^2+y^2+z^2}_{g} = \underbrace{24}_{k}$

Solve: $\boxed{\begin{matrix}\nabla f = \lambda \nabla g \\ g = k\end{matrix}}$

$$\begin{aligned} f_x &= \lambda g_x \\ f_y &= \lambda g_y \\ f_z &= \lambda g_z \\ g &= k \end{aligned} \rightarrow \left[\begin{aligned} yz\boxed{e^{xyz}} &= 4x\lambda \\ xze^{xyz} &= 2y\lambda \\ xye^{xyz} &= 2z\lambda \\ 2x^2+y^2+z^2 &= 24 \end{aligned}\right.$$

$$e^{xyz} = \frac{4x\lambda}{yz}$$

$$\left[\begin{aligned} x\cancel{z}\left(\frac{4x\cancel{\lambda}}{y\cancel{z}}\right) &= \cancel{2}y\cancel{\lambda} \\ x\cancel{y}\left(\frac{\cancel{4}x\cancel{\lambda}}{\cancel{y}z}\right) &= \cancel{2}z\cancel{\lambda} \\ 2x^2+y^2+z^2 &= 24 \end{aligned}\right.$$

$$\left[\begin{aligned} 2x^2 &= y^2 \\ 2x^2 &= z^2 \\ 2x^2+y^2+z^2 &= 24 \end{aligned}\right] \rightarrow \begin{aligned} 6x^2 &= 24 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

other solve ideas . . .

$$\text{eq 4 eqs}\begin{cases} yze^{xyz} = 4x\lambda \Rightarrow \lambda = \boxed{\dfrac{yze^{xyz}}{4x}} \\ xze^{xyz} = 2y\lambda \\ xye^{xyz} = 2z\lambda \\ 2x^2+y^2+z^2 = 24 \end{cases}$$

3 eqs

$$\begin{cases} \cancel{x}z\,\cancel{e^{xyz}} = 2y\left(\dfrac{y\cancel{z}\,\cancel{e^{xyz}}}{4x}\right) & \cancel{z\neq 0} \\ x\cancel{y}\,\cancel{e^{xyz}} = 2z\left(\dfrac{\cancel{y}z\,\cancel{e^{xyz}}}{4x}\right) & y\neq 0 \\ 2x^2+y^2+z^2=24 \end{cases}$$

$x=1$

$(x)^2 = (1)^2$

$x^2 = 1$

$x=1$ $\quad \cancel{x=-1}$