MATH 344 ... EXAM 2

0) Exam Start Time:

Key

0) Name:

0) MyWSUid:

1) Find all first partial derivative for the given function (do not simplify your answers).

$$f(x,y,z) = \frac{x}{yz} + y\cos(xz) = x y^{-1} z^{-1} + y\cos(xz)$$

$$f_x = y^{-1}z^{-1} - yz\sin(xz) = \frac{1}{yz} - yz\sin(xz)$$

$$f_y = -xy^{-2}z^{-1} + \cos(xz) = \frac{-x}{y^2 z} + \cos(xz)$$

$$f_z = -xy^{-1}z^{-2} - yx\sin(xz) = \text{etc}$$

2) Find f_{xy} and f_{yy} for the given function (simplify your answers).

$$f(x, y) = \tan(xy^2 + 1)$$

$$f_x = \sec^2(xy^2+1)\,[y^2] = (y^2)(\sec^2(xy^2+1))$$

$$f_y = \sec^2(xy^2+1)\,[2xy] = (2xy)(\sec^2(xy^2+1))$$

$$f_{xy} = (2y)\sec^2(xy^2+1) + (y^2)\,2\sec^1(xy^2+1)\sec(xy^2+1)\tan(xy^2+1)\,[2xy]$$

$$\boxed{f_{xy} = 2y\sec^2(xy^2+1) + 4xy^3\sec^2(xy^2+1)\tan(xy^2+1)}$$

$$f_{yy} = (2x)\sec^2(xy^2+1) + (2xy)\left[2\sec^2(xy^2+1)\tan(xy^2+1)\,[2xy]\right]$$

$$\boxed{f_{yy} = 2x\sec^2(xy^2+1) + 8x^2y^2\sec^2(xy^2+1)\tan(xy^2+1)}$$

3) Find a linearization for $f(x,y) = 5xe^{xy}$ at the point $\boxed{(1,0)}$. $f \approx f(1,0) + f_x(1,0)(x-1) + f_y(1,0)(y-0)$

$f_x = 5e^{xy} + 5xye^{xy}$

$f_y = 5x^2e^{xy}$

$f_x(1,0) = 5$

$f_y(1,0) = 5$

$f(1,0) = 5$

$f(x,y) \approx 5 + 5(x-1) + 5(y-0)$

$\boxed{f(x,y) \approx 5x + 5y}$

$\frac{\Delta C}{C} \approx \frac{dC}{C}$

4) Approximate the relative percentage error in calculating the cost of producing a widget if the cost function is given by ...

$$C(x,y,z) = 3x^{1/2} + xy + yz^2$$

@ $(x,y,z) = (4,2,2)$

and the values you measure for x, y, and z are $x = 4 \pm 0.1$, $y = 2 \pm 0.2$, and $z = 2 \pm 0.1$.

$C_x = \frac{3}{2}x^{-1/2} + y$ $\rightarrow C(4,2,2) = 3 \cdot 2 + 4 \cdot 2 + 2 \cdot 2^2 = 22$

$C_y = x + z^2$ $\quad C_x(4,2,2) = \frac{3}{2}\frac{1}{\sqrt{4}} + 2 = \frac{3}{4} + 2 = \frac{11}{4}$

$C_z = 2yz$ $\quad C_y(4,2,2) = 4 + 2^2 = 8$

$C_z(4,2,2) = 2 \cdot 2 \cdot 2 = 8$

$$\frac{\Delta C}{C} \approx \frac{dC}{C} = \frac{C_x\,dx + C_y\,dy + C_z\,dz}{C}$$

$$= \frac{11/4(.1) + 8(.2) + 8(.1)}{22} = \frac{107}{880} \approx 0.12$$

$\approx \boxed{12\%}$

5) Use the chain rule to calculate $\frac{dC}{dt}$ at $\boxed{t = 1}$ for

$$C(x, y, z) = 3x^{1/2} + xy + yz^2$$

where $x(t) = t^2 + t$, $y(t) = 3t + 1$, and $z(t) = 4t$

$C_x = \frac{3}{2}x^{-1/2} + y$ $\qquad x' = 2t + 1$

$C_y = x + z^2$ $\qquad y' = 3$

$C_z = 2yz$ $\qquad z' = 4$

$\boxed{@\, t = 1}$

$x = 2 \quad x' = 3 \quad C_x = 4 + \frac{3}{2\sqrt{2}}$

$y = 4 \quad y' = 3 \quad C_y = 18$

$z = 4 \quad z' = 4 \quad C_z = 32$

$$\frac{dC}{dt} = C_x x' + C_y y' + C_z z'$$

$$= \left(\frac{3}{2}x^{-1/2} + y\right)(2t+1) + (x + z^2)(3) + (2yz)(4)$$

$$= \boxed{\left(4 + \frac{3}{2\sqrt{2}}\right)(3) + (18)(3) + (32)(4)}$$

6) Use the chain rule to calculate the first partial derivative with respect to s for

$$f(x, y) = \underline{3x^{1/2}} + \underline{(xy)^{1/2}} + \underline{y^2}$$

where $x(s, t) = t^2 + 3st$ and $y(s, t) = t^2 + s^2$.

$f_x = \frac{3}{2}x^{-1/2} + \frac{1}{2}y(xy)^{-1/2}$ $\qquad \frac{\partial x}{\partial s} = x_s = 3t$

$f_y = \frac{1}{2}x(xy)^{-1/2} + 2y$ $\qquad \frac{\partial y}{\partial s} = y_s = 2s$

$$\frac{\partial f}{\partial s} = f_x x_s + f_y y_s$$

$$= \boxed{\left(\frac{3}{2}x^{-1/2} + \frac{1}{2}y(xy)^{-1/2}\right)(3t) + \left(\frac{1}{2}x(xy)^{-1/2} + 2y\right)(2s)}$$

7) Find the directional derivative of $f(x, y) = (xy)^{1/2}$ at the point (9,9) and in the direction from (9,9) to (12,5). Note make sure to use the unit vector when calculating the directional derivative.

$v_1 = \langle 12-9, 5-9 \rangle = \langle 3, -4 \rangle$ so $u_1 = \frac{1}{5}\langle 3,-4\rangle = \langle \frac{3}{5}, -\frac{4}{5} \rangle$

$\nabla f = \langle \frac{1}{2}y(xy)^{-1/2}, \frac{1}{2}x(xy)^{-1/2} \rangle = \frac{1}{2\sqrt{xy}}\langle y, x\rangle = \langle \frac{y}{2\sqrt{xy}}, \frac{x}{2\sqrt{xy}} \rangle$

$D_{u_1}(f) = \frac{1}{10\sqrt{xy}}\langle y, x\rangle \cdot \langle 3, -4\rangle$

$= \boxed{\frac{1}{10\sqrt{xy}}(3y - 4x)}$

8) Find the maximum rate of change of $f(x, y) = 2\sin(xy)$ at the point (0,4) and find the direction in which it occurs. Note: Make sure to use a unit vector when calculating the directional derivative.

$\nabla f = \langle 2y\cos(xy), 2x\cos(xy) \rangle$

$\nabla f(0,4) = \langle 8, 0 \rangle$

so Max $= \|\langle 8, 0\rangle\| = 8$

in directn of $\langle 8,0\rangle$ or $\langle 1, 0\rangle$

9) Find any local extrema or saddle points of $f(x, y) = 2 - 2x + 4y - x^2 - y^2$.

$$\begin{cases} f_x = -2 - 2x = 0 \\ f_y = 4 - 2y = 0 \end{cases} \rightarrow \begin{cases} f_x = 0 \text{ says } x = -1 \\ f_y = 0 \text{ says } y = 2 \end{cases} \rightarrow \boxed{(-1,2)}$$

$f_{xx} = -2$, $f_{yy} = -2$, $f_{xy} = 0$

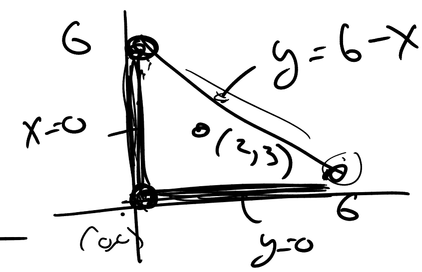
$D = 4 > 0$ and $f_{xx} = -2 < 0$

$$\boxed{\text{So local max @ } (-1,2) \text{ of } f(-1,2) = 7}$$

10) Find the absolute extrema of $f(x, y) = 4x + 6y - x^2 - y^2 + 3$ over the region bounded by $x = 0$, $y = 0$, and $y = 6 - x$.

$f_x = 4 - 2x \rightarrow f_x = 0$ is $x = 2$ $\boxed{(2,3)}$

$f_y = 6 - 2y \rightarrow f_y = 0$ is $y = 3$



$x=0$ $f(y) = 6y - y^2 + 3$ $f' = 6 - 2y$ $y = 3$ $(0,3)$

$f'' = -2$ max

$(0,0)(0,6)$

$y=0$ $f(x) = 4x - x^2 + 3$ $f' = 4 - 2x$ $x = 2$ $(2,0)$

$f'' = -2$ max

$(0,0)$ $(6,0)$

$y = 6-x$ $f(x) = 4x + 6(6-x) - x^2 - (6-x)^2 + 3 = -2x^2 + 10x + 3$

$f' = -4x + 10 \rightarrow x = 5/2$ $(5/2, 7/2)$

$f'' = -4$ max

$f(x,y) = 4x + 6y - x^2 - y^2 + 3$

$$\boxed{\begin{array}{l} \text{Min @ } (6,0) \text{ of } -9 \\ \text{max @ } (2,3) \text{ of } 16 \end{array}}$$

$f_x = 4x - 4 \quad f_y = 6y$

11) Use Lagrange multipliers to find the extreme values of $f(x, y) = 2x^2 + 3y^2 - 4x - 7$ subject to the constraint $x^2 + x^2 = 16$.

Ver 1 $\quad g = 2x^2 \quad g_x = 4x \quad g_y = 0$

$$\begin{cases} 4x - 4 = \lambda(4x) \\ 6y = 0 \quad y = 0 \\ 2x^2 = 16 \end{cases}$$

$x = \pm\sqrt{8}$

$(-\sqrt{8}, 0) \qquad (\sqrt{8}, 0)$

$f(-\sqrt{8}, 0) = \boxed{16 + 4\sqrt{8}}$ Max $\qquad f(\sqrt{8}, 0) = \boxed{16 - 4\sqrt{8}}$ Min

Ver 2 $\quad g(x,y) = x^2 + y^2$

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12) Setup the sytem of equations that would attempt to solve the problem of using Lagrange multipliers to find the extreme values of $f(x, y, z) = e^{xyz}$ subject to the constraint $x^2 + y^2 + z^2 = 1$.

Solve

$$\begin{cases} yz\,e^{xyz} = 2\lambda x \\ xz\,e^{xyz} = 2\lambda y \\ xy\,e^{xyz} = 2\lambda z \\ x^2 + y^2 + z^2 = 1 \end{cases}$$

0) What is the time you ended working on the exam and started scanning it?

#11

$\boxed{g = x^2 + y^2}$ $g_x = 2x$ $g_y = 2y$

$$\begin{cases} 4x - 4 = \lambda(2x) \\ 6y = \lambda(2y) \Rightarrow \text{if } y \neq 0 \\ x^2 + y^2 = 16 \end{cases}$$

$$\begin{cases} 4x - 4 = 2\lambda x \quad 6x \\ 6 = 2\lambda \rightarrow \lambda = 3 \\ x^2 + y^2 = 16 \end{cases}$$

$x = -2$ $y = \pm 2\sqrt{3}$

$(-2, 2\sqrt{3})$, $(-2, -2\sqrt{3})$

but $y = 0$

$$\begin{cases} 4x - 4 = 2\lambda x \\ x^2 = 16 \\ x = \pm 4 \end{cases}$$

so $(4, 0)$, $(-4, 0)$

$f(x,y) = 2x^2 + 3y^2 - 4x - 7$

$f(-2, 2\sqrt{3}) = 45$
$f(-2, -2\sqrt{3}) = 45$ Max
$f(4, 0) = 9$ ← Min

$f(-4, 0) = 41$