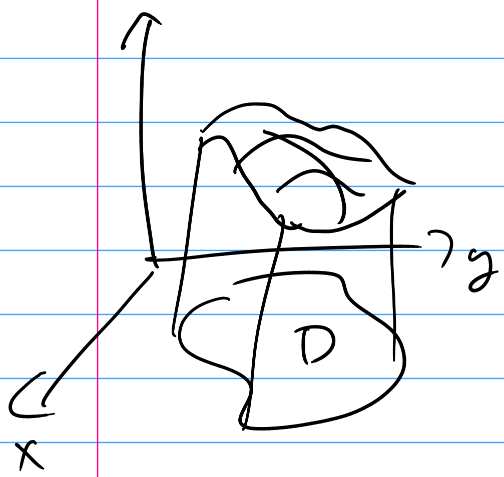
Math 314

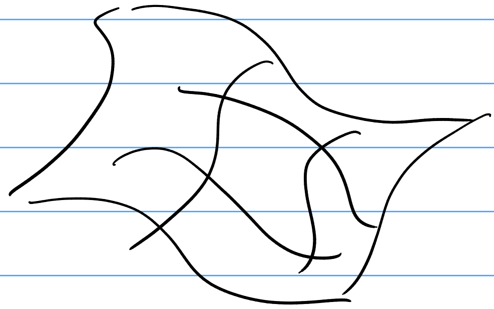


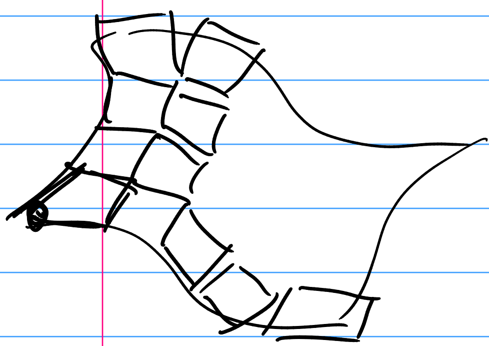
$$Vol = \iint_D f \, dA$$

adding

(height)(area) = vol.

Surface Area



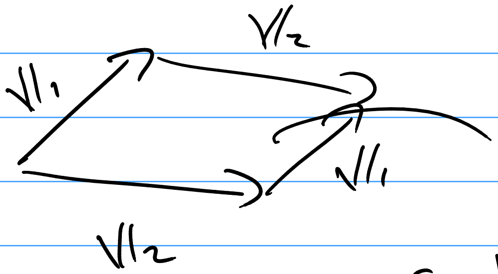


Surface area $\approx$ Sum of small rectangles of tangent planes at points on surface

$$\text{Surface Area} = \iint_D$$

areas of small parts of tangent planes ? ? ?

#1



area $= |v_1 \times v_2|$

#2

in direction of f_x

$$a_1 = \langle 1, 0, f_x \rangle \Delta x$$

$$b = \langle 0, 1, f_y \rangle \Delta y$$

together

area of small part of tangent plane

is $|\mathbf{a} \times \mathbf{b}| = |(\langle 1,0,f_x\rangle \times \langle 0,1,f_y\rangle)\Delta x \Delta y|$

$$= |\langle 1,0,f_x\rangle \times \langle 0,1,f_y\rangle| \underbrace{\Delta x \Delta y}_{\Delta A}$$

$$\left\| \begin{vmatrix} i & j & k \\ 1 & 0 & f_x \\ 0 & 1 & f_y \end{vmatrix} \right\| = |\langle -f_x, -f_y, 1\rangle|$$

$$\Rightarrow \boxed{|\mathbf{a} \times \mathbf{b}|} = \sqrt{1 + f_x^2 + f_y^2}\ \underbrace{\Delta x \Delta y}_{\text{"}\Delta A}$$

So Surface Area Formula $= \iint_D \sqrt{1 + f_x^2 + f_y^2}\, dA$

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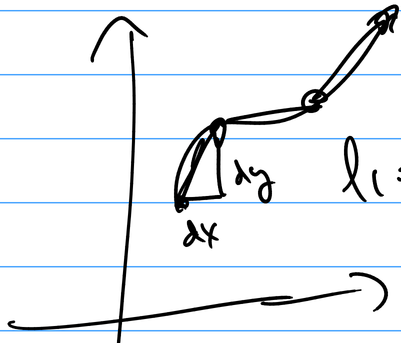
calc 2

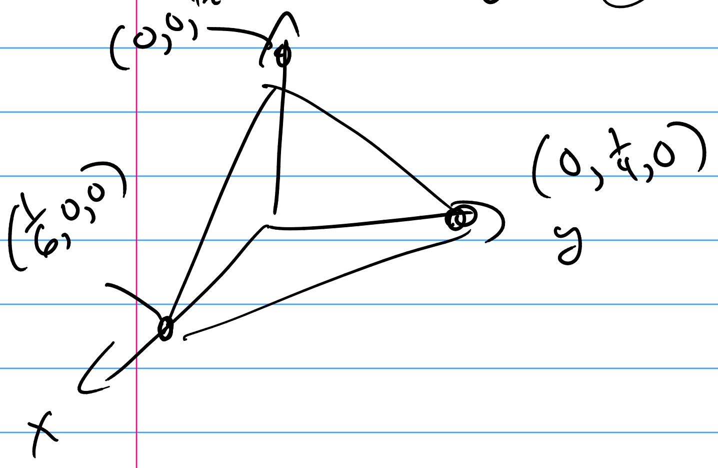


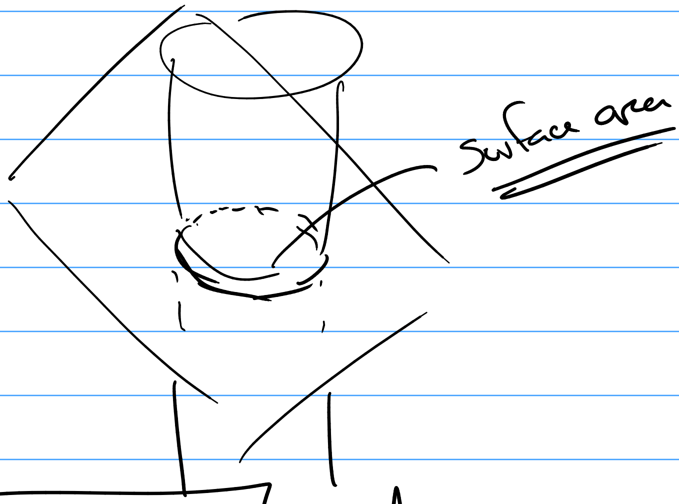
$l_1 = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

arc length $= \int_a^b \sqrt{1 + (f')^2}\, dx$

$z = \frac{1}{2} - 3x - 2y$

Ex) $6x + 4y + 2z = 1$ inside $x^2 + y^2 = 25$

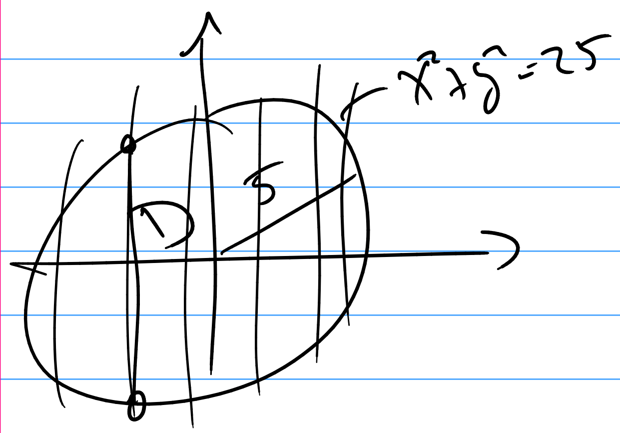




Formula:
$$SA = \iint_D \sqrt{1 + (f_x)^2 + (f_y)^2}\, dA$$

$$z = \frac{1}{2} - 3x - 2y$$
$$f_x = -3$$
$$f_y = -2$$

$$SA = \iint_D \sqrt{14}\, dA$$



$$SA = \sqrt{14} \iint_D 1\, dA$$

$$SA = \sqrt{14} \int_{x=-5}^{5} \left(\int_{y=-\sqrt{25-x^2}}^{y=\sqrt{25-x^2}} 1\, dy \right) dx$$

$$\boxed{SA = \sqrt{14}\left(\pi (5)^2\right)}$$

$z = xy + x^2 + y^2 \qquad 0 \le x \le 2 \qquad 0 \le y \le 2$

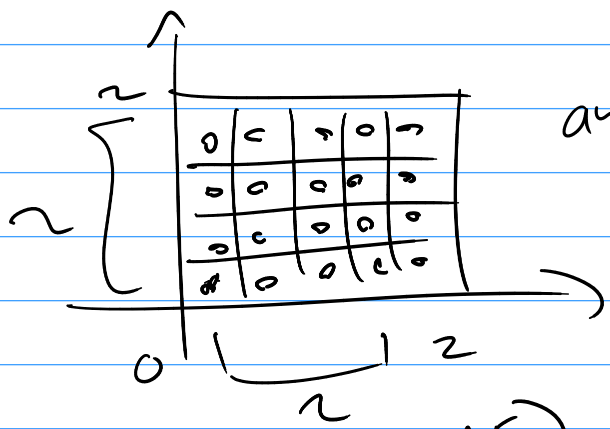
$$SA = \iint_D \sqrt{1 + (f_x)^2 + (f_y)^2}\, dA$$

$$f_x = y + 2x \ , \quad f_y = x + 2y$$

$$SA = \iint_D \sqrt{1 + (y+2x)^2 + (x+2y)^2}\, dA$$

$$SA = \boxed{\iint_D \sqrt{1 + 5y^2 + 5x^2 + 8xy}\, dA}$$

$$SA = \int_0^2 \left[\int_0^2 \sqrt{5x^2 + 8yx + (1+5y^2)}\, dx \right] dy$$



ans $\approx$ (4) average(F(samples))