Math 344

today: Review

Friday: Tech and Calculus

Q's

Monday: Exam 3 opens

Exam 3

12 probs @ 10 pts

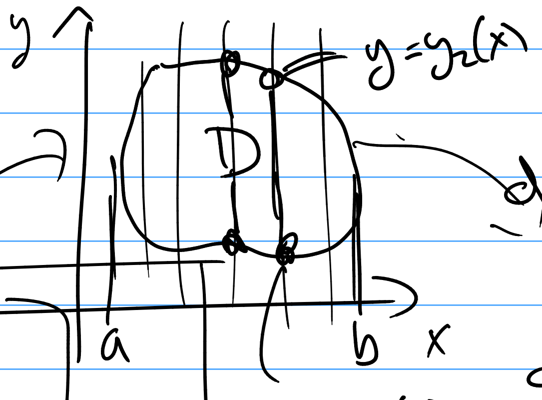
110 pts = 100%

15.1 / 15.2

$$\iint_D f\,dA \qquad dA = dx\,dy$$

3 probs

1

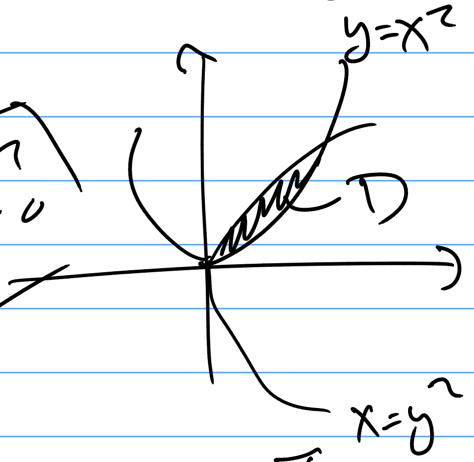


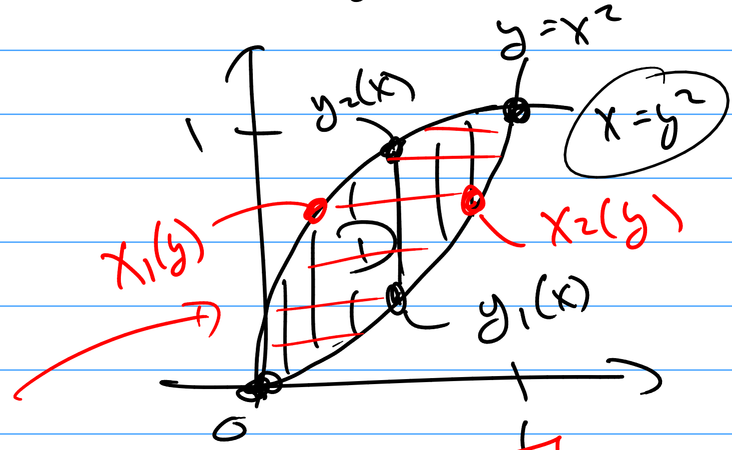
$$\iint_D f\,dA = \int_{x=a}^{x=b}\left[\int_{y=y_1(x)}^{y=y_2(x)} \boxed{f}\,dy\right]dx$$

$$\int_{y=c}^{y=d}\left[\int_{x=x_1(y)}^{x=x_2(y)} \boxed{f}\,dx\right]dy$$

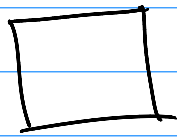
ex D is region between $x=y^2$ and $y=x^2$

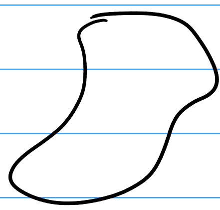
$$\boxed{\iint_D f\,dA = ?}$$





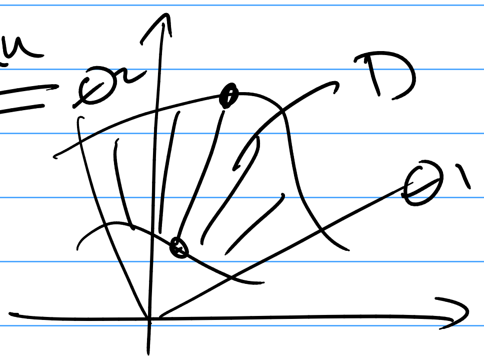
$$\int_{x=0}^{x=1}\left[\int_{y=x^2}^{y=\sqrt{x}} f\,dy\right]dx = \int_{y=0}^{y=1}\left[\int_{x=y^2}^{x=\sqrt{y}} f\,dx\right]dy$$

(2) $\iint_D f\,dA$ but D is 

(3) $\iint_D f\,dA$ but D is 

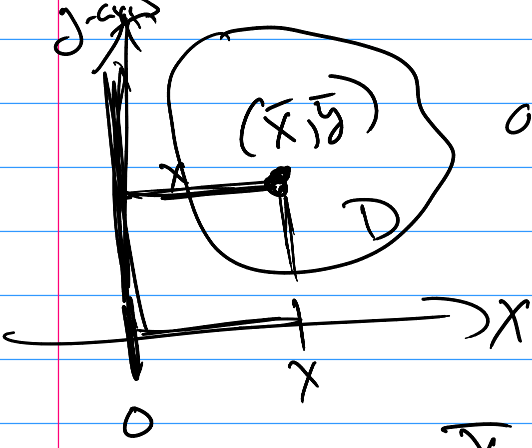
15.3 $\iint_D f\,dA$ but $dA = r\,dr\,d\theta$ (polar)

1 problem



15.4 / 15.5 Applications of $\iint_D f\,dA$ — 2 probs

(1) center of mass

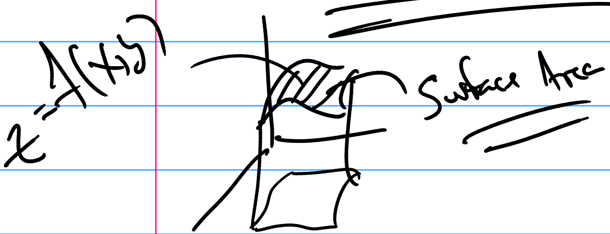


on D $\quad \rho(x,y) = \frac{\text{mass}}{\text{area}}$ (density)

$$M = \iint_D \rho\, dA$$

$$\bar{x} = \frac{\boxed{\iint_D x\rho\, dA}}{M} \qquad \bar{y} = \frac{\iint_D y\rho\, dA}{M}$$

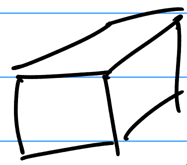
(2) Surface Area



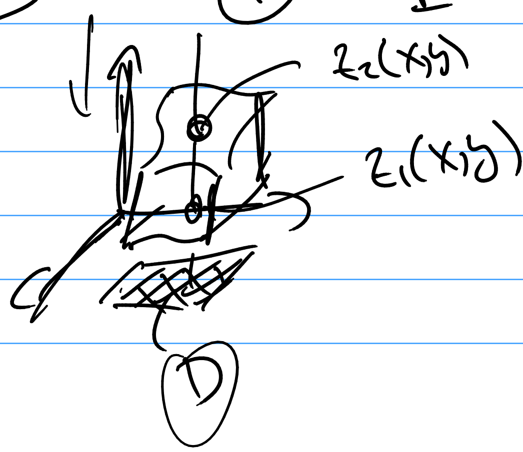
$$SA = \iint_D \sqrt{1 + (f_x)^2 + (f_y)^2}\, dA$$

15.6 $\iiint_E f\, dV \qquad dV = dx\,dy\,dz$

2 probs

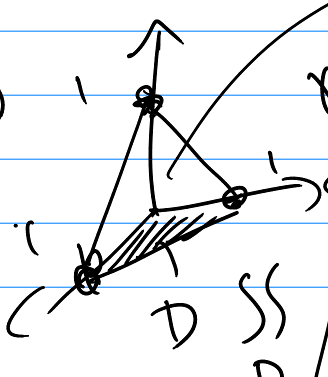
(1) E is 

(2)



$z = 1-x-y$

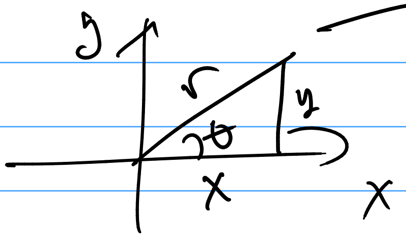
$x+y+z = 1$



$$\iint_D \left[\int_{z=0}^{z=1-x-y} f\, dz\right] dA$$

$\boxed{15.7}$ $dV = r\,dz\,dr\,d\theta$ (cylindrical)

1 pnts



$$\boxed{\begin{aligned} x &= r\cos\theta & x^2+y^2 &= r^2 \\ y &= r\sin\theta \\ z &= z \end{aligned}}$$

$\boxed{15.8}$ Spherical $dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta \qquad x^2+y^2+z^2 = \rho^2$$
$$z = \rho \cos\phi$$

$\boxed{15.9}$ $\iiint_E f\,dV$ w/ $f(x,y,z)$ E is x,y,z functions

$$\begin{aligned} x &= x(u,v,w) \\ y &= y(u,v,w) \\ z &= z(u,v,w) \end{aligned}$$

$$\iiint_S f(x(u,v,w), y(u,v,w), z(u,v,w)) \left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| du\,dv\,dw$$

Jacobian

2D $$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} \checkmark$$

3D $$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix} \checkmark$$