Math 344

Calculus ← keys = functions

ch 14/15 functions: $\boxed{f(x,y)}$, $\boxed{f(x,y,z)}$, ..

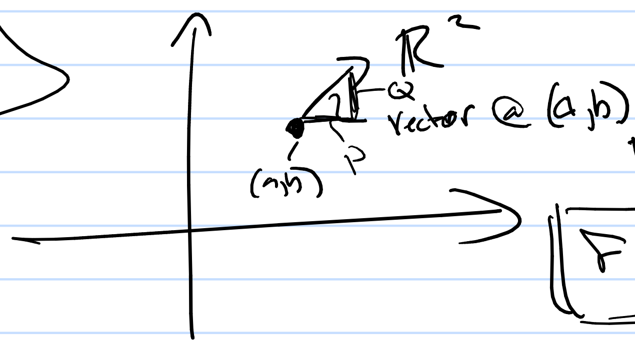
$f(x_1, x_2, .., x_n)$

ch16 Vector Calculus

Functions: Domain $\mathbb{R}^2, \mathbb{R}^3, ..$ $\xrightarrow{\mathbb{F}}$ Codomain Vectors in $\mathbb{R}^2, \mathbb{R}^3, ..$

ex 3D (x,y,z)

Visualize



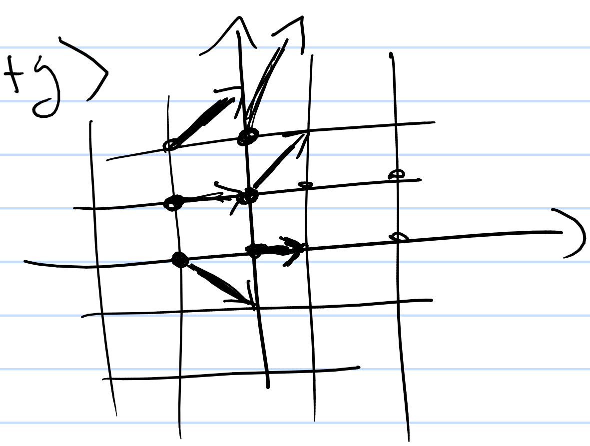
$\mathbb{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$

$\boxed{\mathbb{F} \text{ is a vector field}}$

2D $\mathbb{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$

3D $\mathbb{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$

ex $\mathbb{F} = \langle 1, x+y \rangle$



<u>Why?</u>: $F = \langle P, Q \rangle$ 2D?

$F = \langle P, Q, R \rangle$ 3D?

(1) $\boxed{\text{Vector fields \& forces}}$ (Visualize)

<u>Force fields</u>

(2) <u>Gradient fields</u>

$$\nabla f = \langle f_x, f_y \rangle \quad 2D$$

$$\nabla f = \langle f_x, f_y, f_z \rangle \quad 3D$$

so $f = f(x,y,z)$ so f_x, f_y, f_z are still functions of x, y, z

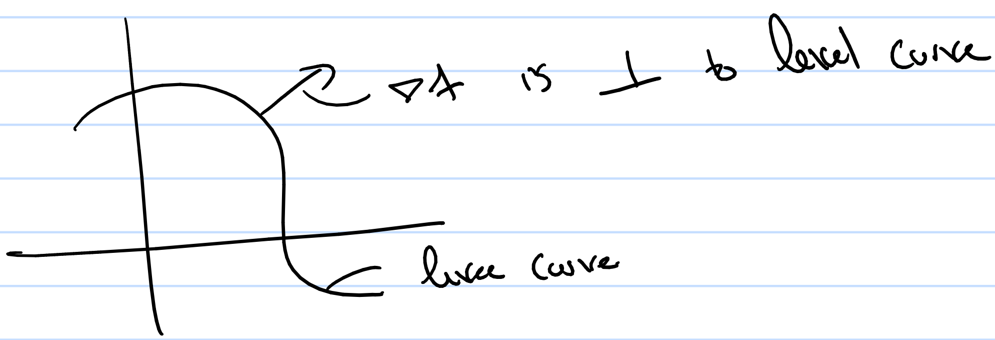
$\Rightarrow \therefore \nabla f$ is a vector field

(ex) $f(x,y,z) = xy + z^2$

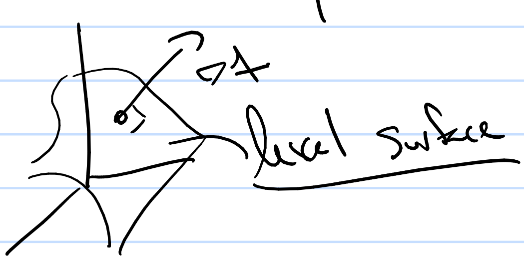
$$\nabla f = \langle y, x, 2z \rangle$$

<u>Note</u>:

2D

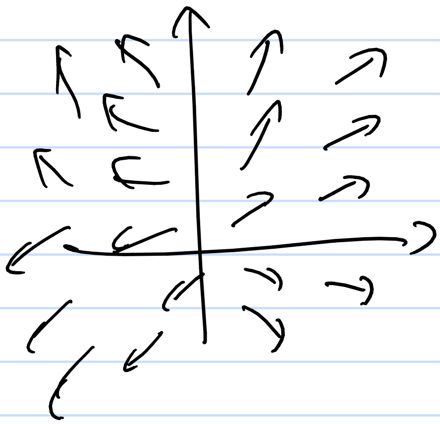


3D



Def if you have a vector field $\mathbf{F}$ and you find $\mathbf{F} = \nabla f$

2D



$\mathbf{F}$ does $f(x,y,z)$ exist such that

$$\nabla f \overset{?}{=} \mathbf{F}$$

if true .. call $\mathbf{F}$ a conservative vector field

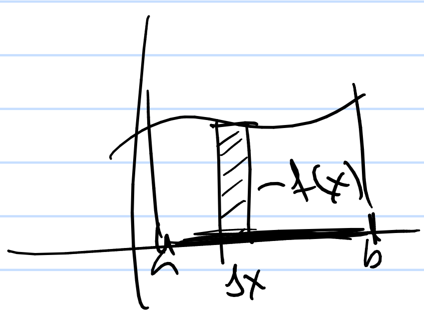
and f is its potential function

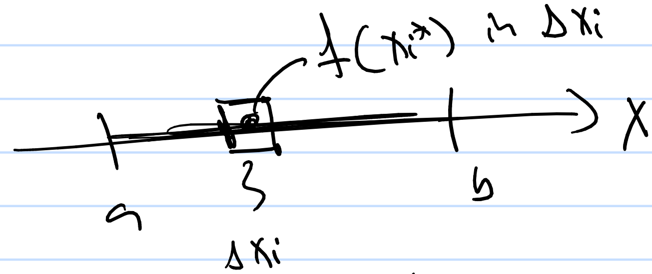
16.2 (Tools for Vector Calculus)

#1 line integral

Calc I

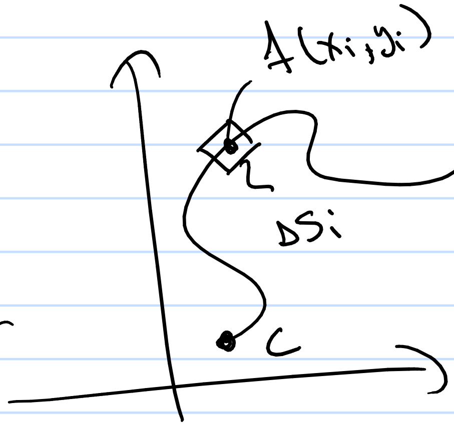
$$\int_a^b f(x)\,dx = \lim \sum \boxed{f(x_i^*)}\boxed{\Delta x_i}$$





adding $(f(x_i^*)\Delta x_i)$

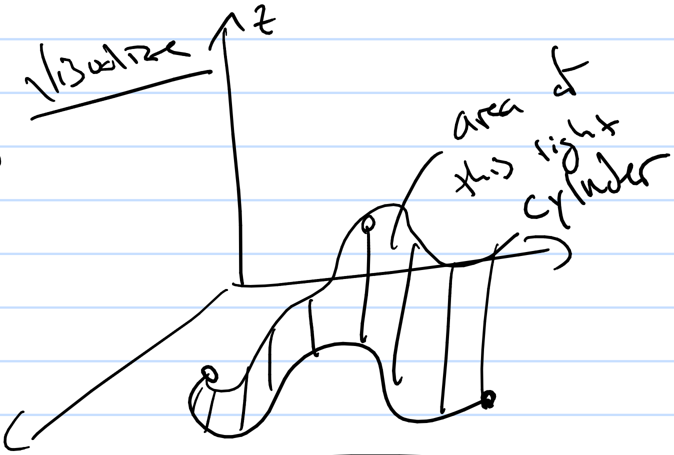
Do same (addition) Integration along a space curve?



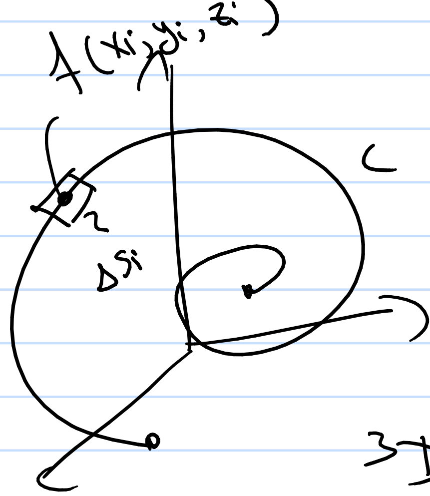
2D

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*, y_i^*) \Delta S_i$$

Visualize



$$\boxed{\int_C f(x,y)\,ds}$$



3D

$$\lim_{n\to\infty} \sum f(x_i^*, y_i^*, z_i^*) \Delta S_i$$

$$\boxed{\int_C f(x,y,z)\,ds}$$

are **line integrals**

to do this we need parametric representations for C

So.. review chapter 10!!

2D if C is $x(t), y(t)$ $a \le t \le b$

3D if C is $x(t), y(t), z(t)$ $a \le t \le b$

So $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$ 2D

and $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$ 3D

2D $\int_C f(x,y)\,ds = \int_{t=a}^{t=b} f(x(t), y(t)) \sqrt{(x')^2 + (y')^2}\,dt$

3D $\int_C f(x,y,z)\,ds = \int_{t=a}^{t=b} f(x(t), y(t), z(t)) \sqrt{(x')^2 + (y')^2 + (z')^2}\,dt$

Notation: parametriz rep. is written $r(t)$

$$\boxed{\int_C f\,ds = \int_{t=a}^{t=b} f(r(t))\,|r'(t)|\,dt}$$