Math 344 ... Exam 3

0) Exam Start Time:

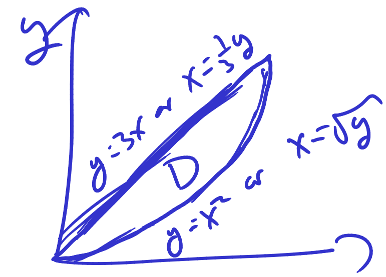
0) Name: Key

0) MyWSUid:

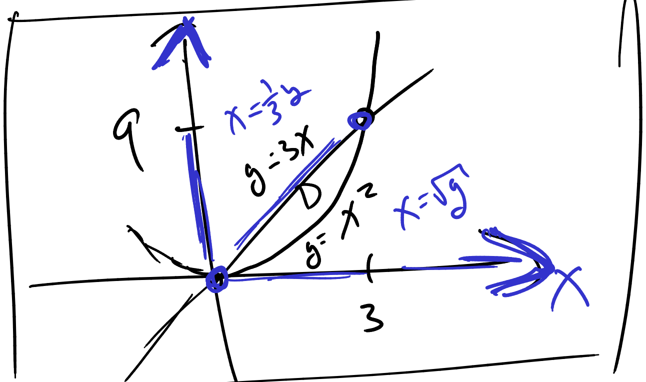
$x^2 = 3x$

$x^2 - 3x = 0$

$x(x-3) = 0$



1) Setup the integral of $(xy + y^2)$ over the region bounded by $y = 3x$ and $y = x^2$ in the order of $dy\,dx$ and in the order of $dx\,dy$. (DO NOT INTEGRATE)



$y = 3x \rightarrow x = \frac{1}{3}y$

$y = x^2 \rightarrow x = \sqrt{y}$

$$\int_{x=0}^{x=3} \int_{y=x^2}^{y=3x} (xy + y^2)\, dy\, dx = \int_{y=0}^{y=9} \int_{x=\frac{1}{3}y}^{x=\sqrt{y}} (xy + y^2)\, dx\, dy$$

2) Evaluate the given double integral by hand. Show all work.

$$\int_{-1}^{2}\int_{0}^{2}(xy+y^2)\,dy\,dx = \int_{-1}^{2} \frac{1}{2}xy^2+\frac{1}{3}y^3\Big|_0^2\,dx$$

$$= \int_{-1}^{2}\left(2x+8/3\right)dx = x^2+\frac{8}{3}x\Big|_{-1}^{2}$$

$$= \left(4+\frac{16}{3}\right)-\left(1-\frac{8}{3}\right) = 3+8 = \boxed{11}$$

3) Evaluate the given double integral by hand. Show all work.

$$\int_0^1\int_{x^2}^{3x}\frac{x}{(xy+1)^2}\,dy\,dx \qquad u = xy+1 \quad du = x\,dy$$

$$\rightarrow \int_{x=0}^{x=1}\left(\int_{u=x^3+1}^{u=3x^2+1} u^{-2}\,du\right)dx = \int_{x=0}^{x=1}\left(-\frac{1}{u}\Big|_{u=x^3+1}^{u=3x^2+1}\right)dx \qquad \frac{1}{x^2+a^2}$$

?

+5

$$= \boxed{\int_{x=0}^{x=1}\left(\frac{1}{x^3+1}-\boxed{\frac{1}{3x^2+1}}\right)dx} = \int_0^1 \boxed{\frac{1}{x^3+1}}dx - \int_0^1\frac{1}{3x^2+1}\,dx$$

+3

$$= \int_0^1 \boxed{\frac{1}{(x+1)(x^2-x+1)}}\,dx - \frac{1}{\sqrt{3}}\tan^{-1}(\sqrt{3}x)\Big|_0^1 = \boxed{\frac{1}{3}\ln(2)}$$

$\sqrt{3}$

by partial fractions

+3

$$\int\frac{\frac{2}{3}-\frac{1}{3}x}{x^2-x+1}\,dx + \int\frac{1/3}{x+1}\,dx$$

lots of scratch work (1/2 page)

$\tan^{-1}$ $\quad$ ln $\quad$ = $\quad$ ln

+3 $\quad$ +3

All right? +17

$$\int \frac{1}{x^3+1} - \frac{1}{3x^2+1}\,dx = \int \frac{1}{(x+1)(x^2-x+1)} - \frac{1}{3x^2+1}\,dx$$

Integration tables for . . .

$$\int \frac{1}{\text{quadratic}} \qquad \int \frac{1}{\text{linear}}$$

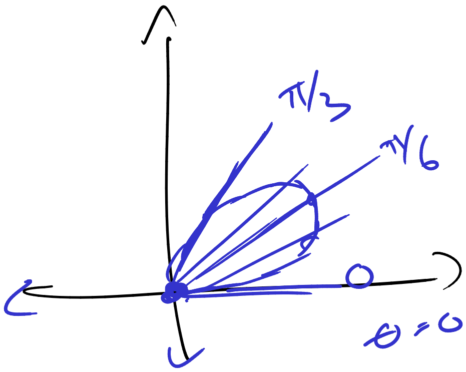
$$\int \frac{\text{linear}}{\text{quadratic}}$$

1)

$$\int \frac{A}{x+1} + \frac{B+Cx}{x^2-x+1} - \frac{1}{3x^2+1}\,dx$$

$$\underbrace{\int \frac{A}{x+1}\,dx}_{①} + \underbrace{\int \frac{B}{x^2-x+1}\,dx}_{②} + \underbrace{\int \frac{Cx}{x^2-x+1}\,dx}_{②} + \underbrace{\int \frac{1}{3x^2+1}\,dx}_{③}$$

4) Setup the double integral in polar form to find the area of one loop of $r = \sin(3\theta)$ (DO NOT INTEGRATE).



$$\boxed{\int_0^{\pi/3}\int_0^{\sin(3\theta)} r\,dr\,d\theta}$$

$$r = \sin(3\cdot 0) = 0$$

$$r = \sin(3(\pi/3)) = \sin(\pi) = 0$$

$\theta = \pi/3$

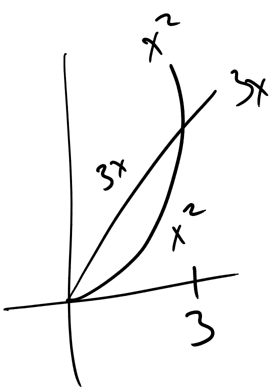
$r = 0$

$\sin(3\theta) = 0$

$\theta = 0$

$\theta = \pi/3$

5) Setup the three double integrals to find the center of mass for a lamina with density function $\rho(x, y) = xy^2$ that occupies the region bounded by $y = 3x$ and $y = x^2$. (DO NOT INTEGRATE)



$$M = \int_0^3\int_{x^2}^{3x} xy^2\,dy\,dx$$

$$\bar{x} = \frac{1}{M}\boxed{\int_0^3\int_{x^2}^{3x} x^2y^2\,dy\,dx}$$

$$\bar{y} = \frac{1}{M}\boxed{\int_0^3\int_{x^2}^{3x} xy^3\,dy\,dx}$$

6) Find the surface area of the given surface $z = 1 + 2y + 3x + 4x^2$, $0 \le x \le 1$, $1 \le y \le 4$ (you do not have to show all work).

$$f_x = 3 + 8x \qquad f_y = 2 \qquad \iint \sqrt{1 + (f_x)^2 + (f_y)^2}\, dA$$

$$\int_0^1 \int_1^4 \sqrt{1 + (3+8x)^2 + (2)^2}\, dy\, dx$$

$$= \int_0^1 \int_1^4 \sqrt{14 + 48x + 64x^2}\, dy\, dx$$

$$= \boxed{\int_0^1 \sqrt{14 + 48x + 64x^2}\, dx} \boxed{\int_1^4 dy}$$

$$\approx \boxed{22.17}$$

7) Evaluate the given triple integral by hand over the rectangular box B bounded by $0 \leq x \leq 1$, $0 \leq y \leq 2$, and $0 \leq z \leq 3$. Show all work.

$$\iiint_B (x+y)z^2 \, dV$$

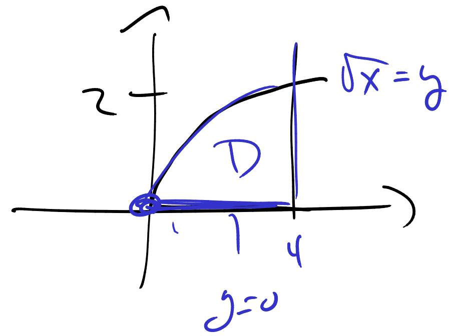
$$\int_0^1 \int_{y=0}^2 \int_{z=0}^3 (x+y)z^2 \, dz \, dy \, dx$$

$$= \int_0^1 \int_0^2 9(x+y) \, dy \, dx$$

$$= 9 \int_0^1 (2x+2) \, dx = 9 \cdot 3 = \boxed{27}$$

8) Evaluate the given triple integral by hand where E lies below the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = x^{1/2}$, $y = 0$, and $x = 4$. Show all work.

$$\int\int\int_E (21xy)\; dV$$



$$\int_{x=0}^{4}\left(\int_{y=0}^{\sqrt{x}}\left(\int_{z=0}^{1+x+y} 21xy\, dz\right) dy\right) dx$$

$$= \int_0^4\left(\int_0^{\sqrt{x}} 21xy(1+x+y)\,dy\right) dx$$

do show more than I did

$$= \int_0^4 \left(\frac{21}{2}x^3 + 7x^{5/2} + \frac{21}{2}x^2\right) dx$$

show a bit more

$$= \boxed{1152}$$

9) Use cylindrical coordinates to evaluate the given triple integral where E is the solid bounded below by $z = \underbrace{(x^2 + y^2)^{1/2}}_{r^2}$ and above by $z = 2$. (you do not need to show all integration work)

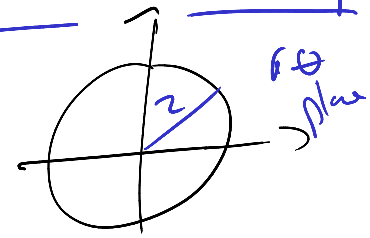
$\boxed{z = r}$

$$\iiint_E \underbrace{(x^2 + y^2)}_{r^2} \, dV$$

and

$z = 2$ $z = r$

$\to \ r = 2$



$$\int_0^{2\pi} \int_0^{2} \int_{z=r}^{z=2} r^2 \, r \, dz \, dr \, d\theta$$

$$= \int_0^{2\pi} \left(\int_0^{2} \left(\int_r^{2} r^3 \, dz \right) dr \right) d\theta$$

$$= \boxed{\frac{16}{5}\pi}$$

10) Setup the triple integral using spherical coordinates where E is the portion of the solid $x^2 + y^2 + z^2 \leq 1$ that lies in the first octant. (DO NOT INTEGRATE).

$$\iiint_E x\, e^{x^2+y^2+z^2}\, dV$$

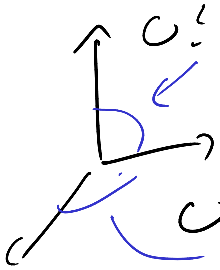
ρ^2

$\rho^2 \leq 1$

or

$\rho \leq 1$

$0 \leq \phi \leq \pi/2$



$0 \leq \theta \leq \pi/2$

$$\boxed{\int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 \rho \sin\phi \cos\theta\, e^{\rho^2} \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi}$$

1st octant.

11) Find the Jacobian for the transformation $x = wu$, $y = vw$, $z = vu$.

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix} = \begin{vmatrix} w & 0 & u \\ 0 & w & v \\ v & u & 0 \end{vmatrix}$$

$$= w \begin{vmatrix} w & v \\ u & 0 \end{vmatrix} + u \begin{vmatrix} 0 & w \\ v & u \end{vmatrix}$$

$$= -uvw - uvw = \boxed{-2uvw}$$

$$|\text{Jacobian}| = 2uvw$$

12) Find the Jacobian for the transformation $x = u\sin(v)$ and $y = u\cos(v)$.

$$\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} \sin v & u\cos v \\ \cos v & -u\sin v \end{vmatrix}$$

$$= -u\sin^2 v - u\cos^2 v$$

$$= \boxed{-u}$$ 4

$$|\text{Jacobian}| = |-u| = \boxed{u}$$ 2

0) What is the time you ended working on the exam and started scanning it?