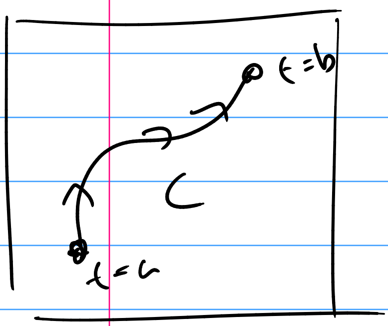
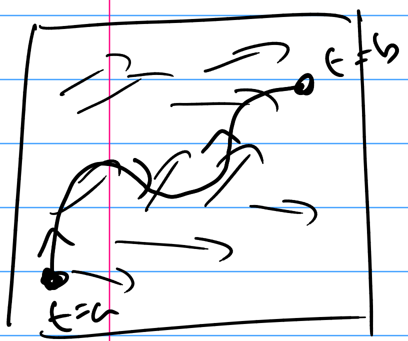
Math 344



Line Integral $\int_C \boxed{f}\, ds$

if C is $r(t)$ $a \le t \le b$

$$\int_C f\, ds = \int_{t=a}^{t=b} f(r(t))\, |r'|\, dt$$



$F = \langle P, Q \rangle$ $\boxed{2D}$

$F = \langle P, Q, R \rangle$ $\boxed{3D}$

Vector fields

Note: if the vectors are forces

$\to$ **Force Field**

Application was work

$$\int_C (F \cdot T)\, ds$$

C is $r(t)$ $a \le t \le b$

$$= \int_a^b \left(\underline{F(r(t)) \cdot r'(t)} \right) dt$$

$$= \int_C F \cdot dr = \int_C P\,dx + Q\,dy \quad \text{2D}$$

on Friday was

$$\int_C \mathbf{F}\cdot d\mathbf{r}$$

if $\boxed{\mathbf{F} = \nabla f}$ ($\mathbf{F}$ conservative vector field)

f is its potential function

then $$\int_C \mathbf{F}\cdot d\mathbf{r} = f(\underbrace{\mathbf{r}(b)}_{\text{end location}}) - f(\underbrace{\mathbf{r}(a)}_{\text{start location}})$$

C is $\mathbf{r}(t)$
$a \le t \le b$

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given $\mathbf{F}$ is it conservative?

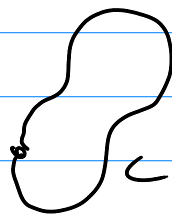
and if it is can you find f so that

$$\nabla f = \mathbf{F}?$$

Note: if $\mathbf{F}$ is conservative then we can show

$\int_C \mathbf{F}\cdot d\mathbf{r}$ is path independent.

Th^m (1) $\int_C \mathbf{F}\cdot d\mathbf{r}$ on a closed curve



$\int_C \mathbf{F}\cdot d\mathbf{r} = 0$ means ind. of path.

(2) Ind. of path $\rightarrow$ $\mathbf{F}$ is conservative

2D

Th^n if $\mathbb{F} = \langle P, Q \rangle$ and $\mathbb{F}$ is conservative

then $\underline{P_y} = Q_x$

Concept? (1) <u>Conserv.</u> $\mathbb{F} = \nabla f = \langle f_x, f_y \rangle$

(2) $\mathbb{F} = \langle P, Q \rangle$ $\qquad P = f_x \qquad Q = f_y$

$\nabla f = \langle f_x, f_y \rangle$ $\qquad P_y = f_{xy} \qquad Q_x = f_{yx}$

Th^n if $\mathbb{F} = \langle P, Q \rangle$ on an open simply-connected domain D

does not contain boundry

no holes

and P, Q have continuous 1^{st} order partials

and $P_y = Q_x$ <u>then</u> $\mathbb{F}$ is conserv

with $\mathbb{F} = \nabla f = \langle f_x, f_y \rangle$, where $f_x = P$, $f_y = Q$

So $\int_c \mathbb{F} \cdot d\mathbf{r}$ on $\qquad \mathbb{F} = \langle x^2 y^3, x^3 y^2 \rangle$

$\mathbf{r} = \langle \underbrace{t^3 - 2t}_{x}, \underbrace{t^3 + 2t}_{y} \rangle$

<u>w/o Fundamental Th^n</u> $\qquad 0 \le t \le 1$

$$\int_c \mathbb{F} \cdot d\mathbf{r} = \int_0^1 \left(\langle (t^3-2t)^2 (t^3+2t)^3, (t^3-2t)^3 (t^3+2t)^2 \rangle \cdot \langle 3t^2-2, 3t^2+2 \rangle \right) dt$$

$= \; ? \; =$

f_x? f_y?

VS Fundamental th^n

$\vec{F} = \langle \underset{P}{x^2y^3}, \boxed{\underset{Q}{x^3y^2}} \rangle$

$\vec{r} = \langle t^3 - 2t, t^3 + 2t \rangle$

$0 \le t \le 1$

check $P_y \overset{?}{=} Q_x$

$3x^2y^2 \overset{?}{=} 3x^2y^2$ true

@ $t=0$ $\vec{r}(0) = \langle 0, 0 \rangle$

$t=1$ $\vec{r}(1) = \langle -1, 3 \rangle$

So $\nabla f = \vec{F}$!

So find f knowing

$f_x = x^2y^3$

$f_y = x^3y^2$

$f_x = x^2y^3$ $\quad \int x^2y^3\,dx = \frac{1}{3}x^3y^3 + c(y)$

$f = \frac{1}{3}x^3y^3 + c(y)$

$f_y = x^3y^2 + c'(y)$ and I know $= x^3y^2$

So $c'(y) = 0$

$c(y) = \text{const.}$ let $c = 0$

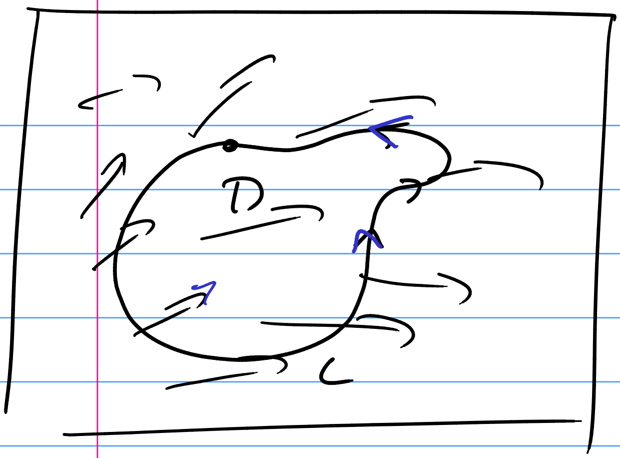
$f(x,y) = \frac{1}{3}x^3y^3$

@ $t=0$ $(0,0)$

@ $t=1$ $(-1,3)$

$$\int_C \vec{F}\cdot d\vec{r} = \frac{1}{3}(-1)^3(3)^3 - \frac{1}{3}(0)^3(0)^3$$

$$= \boxed{-9}$$



$\mathbb{F}$

C is positively oriented, piecewise smooth, simple closed curve

{2D}

$$\oint_C \mathbb{F} \cdot d\mathbf{r} = \oint_C P\,dx + Q\,dy$$

$\mathbb{F} = \langle P, Q \rangle$

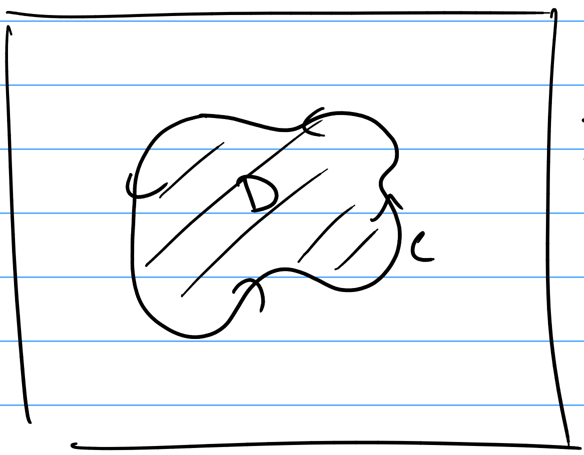
Green's th^n

2D

$$\oint_C \mathbb{F} \cdot d\mathbf{r} = \oint_C P\,dx + Q\,dy$$

(P, Q)

$$= \iint_D (Q_x - P_y)\, dA$$



C is D's boundry

∴ use ∂D for C.