Math 344

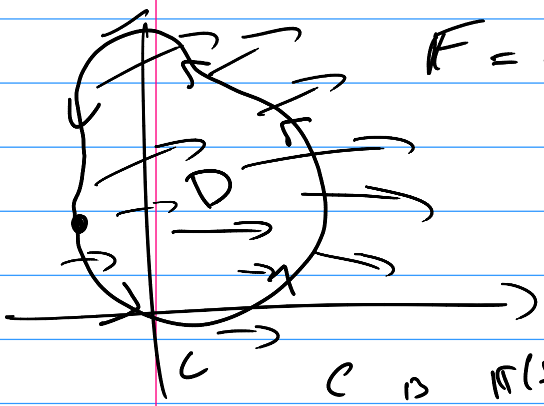
$\boxed{\int_C 1\, ds}$ / $\int_C \mathbf{F}\cdot d\mathbf{r}$

$\int_C (\mathbf{F}\cdot\mathbf{T})\, ds$

C is $\mathbf{r}(t)$

$a \le t \le b$

16.4 Green's th^m (2D)



$\mathbf{F} = \langle P, Q\rangle$

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_D (Q_x - P_y)\, dA$$

C is $\mathbf{r}(t) = \langle x(t), y(t)\rangle$

$a \le t \le b$

Apps of Green's

① $\boxed{\oint_C \mathbf{F}\cdot d\mathbf{r}}$

$\mathbf{F} = \langle P, Q\rangle$

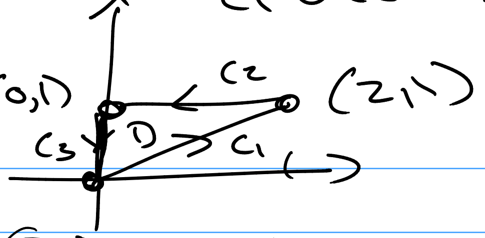
$\mathbf{r}(t) = \langle x(t), y(t)\rangle$

$a \le t \le b$

old way: $\int_a^b (\mathbf{F}(\mathbf{r}(t))\cdot \mathbf{r}')\, dt$

vs green's $\iint_D (Q_x - P_y)\, dA$

C is $C_1 \cup C_2 \cup C_3$



$$\begin{array}{cc} P & Q \end{array}$$

⑨ $\mathbb{F} = \langle x^2+y^2, x^2-y^2 \rangle$

$\boxed{\mathbf{r}(t)\,?}$

$\to C_1 \quad \mathbf{r}_1 = \langle 2t, t \rangle \quad 0 \le t \le 1$

$\to C_2 \quad \mathbf{r}_2 = \langle 2-2t, 1 \rangle \quad 0 \le t \le 1$

$\to C_3 \quad \mathbf{r}_3 = \langle 0, 1-t \rangle \quad 0 \le t \le 1$

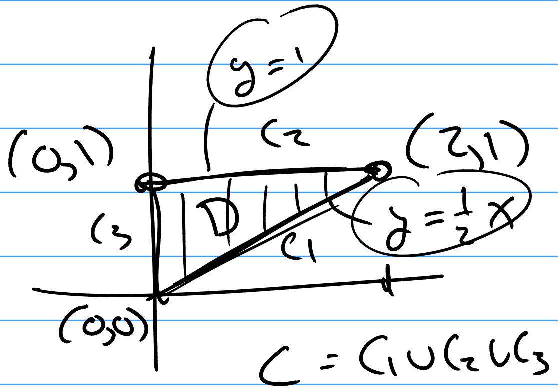
$$\oint_C = \int_{C_1} + \int_{C_2} + \int_{C_3} = \boxed{\text{finish}}$$

$$\int_0^1 \langle (2t)^2+(t)^2, (2t)^2-(t)^2 \rangle \cdot \langle 2,1 \rangle \, dt$$

⑮ using Greens

$$\begin{array}{cc} P & Q \end{array}$$

$\mathbb{F} = \langle x^2+y^2, x^2-y^2 \rangle$



$C = C_1 \cup C_2 \cup C_3$

$$\oint_C \mathbb{F} \cdot d\mathbf{r} = \iint_D (Q_x - P_y)\, dA$$

$$= \iint_D (2x - 2y)\, dA = 2\int_0^2 \left[\int_{\frac{1}{2}x}^1 (x-y)\, dy \right] dx$$

$$= \boxed{\text{finish}}$$

App #2

$$\iint_D (Q_x - P_y)\,dA \overset{?}{=} \oint_C \mathbf{F}\cdot d\mathbf{r}$$

$\boxed{\text{area of } D}$ (using double integral)

$$\iint_D 1\,dA$$

$$\Big\downarrow$$

$$\iint_D \left(\boxed{Q_x - P_y}\right) dA = \oint_C \mathbf{F}\cdot d\mathbf{r}$$

$$\|$$

$$\boxed{\begin{matrix} P = 0 \\ Q = x \end{matrix}}_{\#1} \quad \boxed{\begin{matrix} P = -y \\ Q = 0 \end{matrix}}_{\#2} \quad \boxed{\begin{matrix} P = -\frac{1}{2}y \\ Q = \frac{1}{2}x \end{matrix}}_{\#3} \qquad \oint_C P\,dx + Q\,dy$$

So area of $D = \iint_D (1)\,dA = A$

$$A = \underbrace{\oint_C x\,dy}_{\#1} = \underbrace{-\oint_C y\,dx}_{\#2} = \underbrace{\frac{1}{2}\oint_C x\,dy - y\,dx}_{\#3}$$

$$\boxed{\begin{matrix} C \text{ is } \quad \mathbf{r}(t) = \langle x(t), y(t)\rangle & \\ a \le t \le b & dx = x'\,dt \\ & dy = y'\,dt \end{matrix}}$$

$\boxed{16.5}$ for 3D $\mathbb{F} = \langle P, Q, R \rangle$

borrow ∇ notation of gradient $\nabla f = \langle \frac{\partial}{\partial x} f, \frac{\partial}{\partial y} f, \frac{\partial}{\partial z} f \rangle$

call $\nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$

① curl $\mathbb{F} = \langle P, Q, \textcircled{R} \rangle$

$$\text{curl}(\mathbb{F}) = \boxed{\nabla \times \mathbb{F}} = \begin{array}{ccc} + & - & + \\ \hline i & \textcircled{j} & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{array}$$

$$= \boxed{\langle R_y - \underline{\underline{Q_z}}, P_z - R_x, \underline{\underline{Q_x}} - \underline{P_y} \rangle}$$

② <u>divergence</u>

$$\text{div}(\mathbb{F}) = \nabla \cdot \mathbb{F} = P_x + Q_y + R_z$$