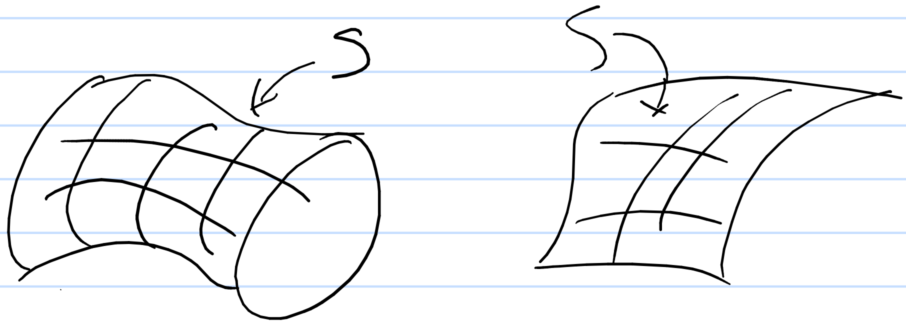
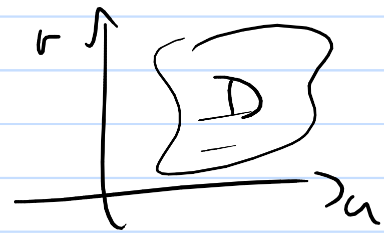
Math 344

Parametric Surface:

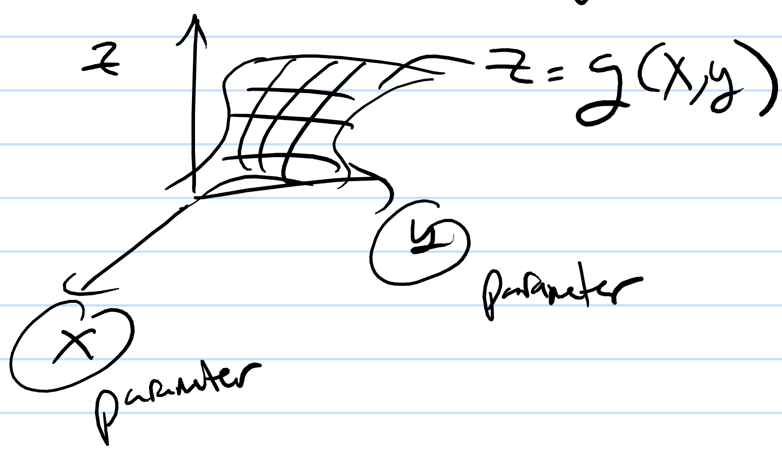


$$r(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$$

u, v are from D



Note: Special case if S has a rectangular function representation



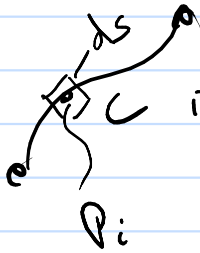
Parametrize?

$$r(x,y) = \langle x, y, g(x,y) \rangle$$

$$r(u,v) = \langle u, v, g(u,v) \rangle$$

$$r(t_1,t_2) = \langle t_1, t_2, g(t_1,t_2) \rangle$$

$\int_C f\,ds$

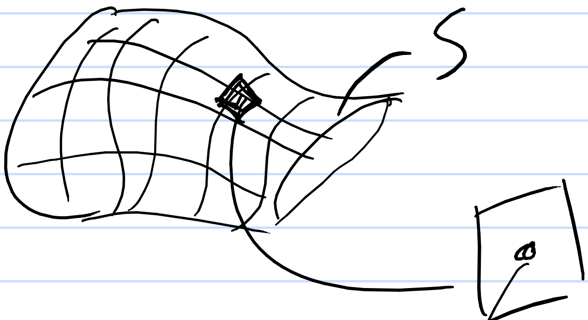


C is $r(t) = \langle x(t), y(t), z(t) \rangle$

$a \le t \le b$

add along C $\boxed{f(p_i)\,ds}$

$$\boxed{\int_a^b f(r(t))\,|r'|\,dt}$$



dS

$$\iint_S f\,dS = \lim_{n\to\infty}\lim_{n\to\infty}\sum_{i=1}^{n}\sum_{j=1}^{n} f(P_{ij})\,\Delta S_{ij}$$

point $P_{ij} = (x_{ij}, y_{ij}, z_{ij})$

if S is $r(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$

over u,v in D

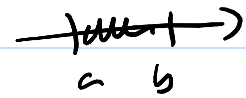
then $\Delta S \approx |r_u \times r_v|\,\Delta u\,\Delta v$

so $dS = |r_u \times r_v|\,\underbrace{du\,dv}_{dA}$

So $$\iint_S f\,dS = \iint_D f(r(u,v))\,|r_u \times r_v|\,dA$$

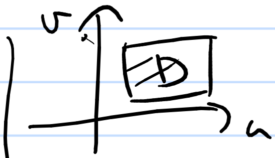
Note: line Integral f over C is $r(t)$ $a \le t \le b$

$$\int_C f\,ds = \int_a^b f(r(t))\,|r'(t)|\,dt$$



Surface Integral f over S is $r(u,v)$ u,v in D

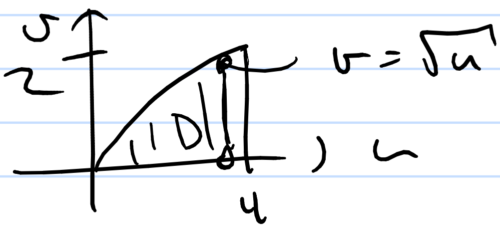
$$\iint_S f\,dS = \iint_D f(r(u,v))\,\boxed{|r_u \times r_v|\,dA}$$



(ex) $f(x,y,z) = x + yz$

S is $r(u,v) = \langle u, uv, u^2+v^2 \rangle$ &

over



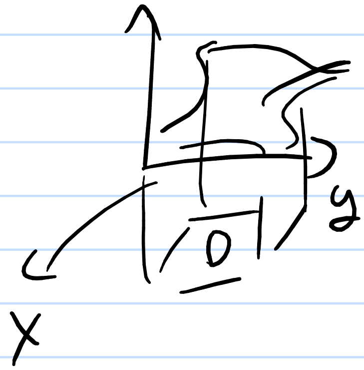
$$\iint_S (x+yz)\,dS = \iint_D (u + uv(u^2+v^2))\,|r_u \times r_v|\,dA$$

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ 1 & v & 2u \\ 0 & u & 2v \end{vmatrix} = \langle 2v^2 - 2u^2, -2v, u \rangle$$

$$|r_u \times r_v| = \sqrt{(2v^2-2u^2)^2 + (-2u)^2 + (u)^2}$$

So $$\iint_S (x+yz)\,dS = \int_0^4 \int_0^{\sqrt{u}} (u + uv(u^2+v^2))\sqrt{(2v^2-2u^2)^2 + (-2u)^2 + (u)^2}\,dv\,du$$

Note: remember S as

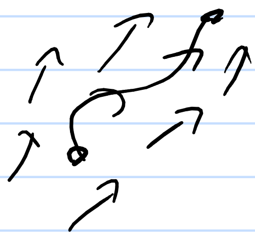


S is $g(x,y)$

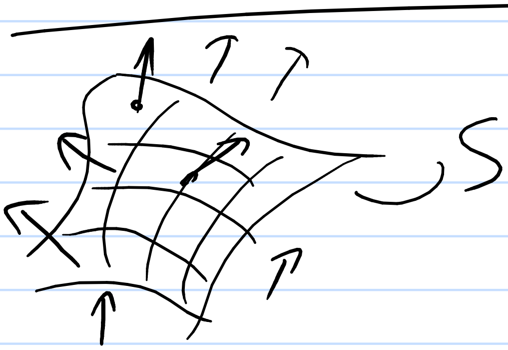
parametric
$\langle x, y, g(x,y) \rangle$

$$\iint_S f \, dS = \iint_D f(x,y,g(x,y)) \sqrt{1 + (g_x)^2 + (g_y)^2} \, dA$$

Line Integral in $\mathbb{F}$



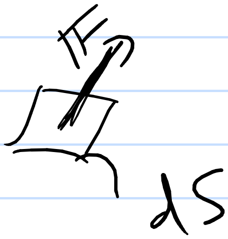
$$\int_C \mathbb{F} \cdot d\mathbf{r} = \int_C (\mathbb{F} \cdot \mathbb{T}) \, ds$$



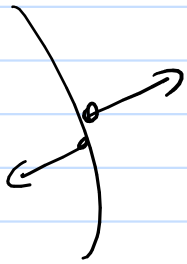
is in $\mathbb{F}$

Application $\mathbb{F}$ is flow of fluid material

$\mathbb{F} = \rho \, \mathbf{v}$ velocity vector field



$\mathbb{F} \cdot \mathbf{n}$



S

S is a boundary of a 3D region E

define $\mathbf{n} = \dfrac{\mathbf{r}_u \times \mathbf{r}_v}{|\mathbf{r}_u \times \mathbf{r}_v|}$

S in $\mathbf{F}$

<u>flux</u> $\displaystyle\iint_S (\mathbf{F} \cdot \mathbf{n})\, dS$ flow through the surface

<u>typical</u> $\mathbf{F} = \rho \mathbf{v}$ fluid flow

density

velocity vector field

$$\iint_S (\rho \mathbf{v} \cdot \mathbf{n})\, dS$$

$\dfrac{\mathbf{r}_u \times \mathbf{r}_v}{|\mathbf{r}_u \times \mathbf{r}_v|}$ $\quad |\mathbf{r}_u \times \mathbf{r}_v|\, dA$

<u>Notation:</u>

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS$$

$$= \iint_D \mathbf{F} \cdot \underline{(\mathbf{r}_u \times \mathbf{r}_v)}\, dA$$