Math 344

$$\boxed{\int_C f\,ds} = \boxed{\int_a^b f(\mathbf{r}(t))\,|\mathbf{r}'(t)|\,dt}$$

line integrals

C is $\mathbf{r}$ ← parametric form of C
over parameter $t \in [a,b]$

Line Integral of interest

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_C \underbrace{\mathbf{F}\cdot\mathbf{T}}_{f}\,ds$$

Surface Integral

$$\iint_S f\,dS = \iint_D f(\mathbf{r}(u,v))\,|\mathbf{r}_u \times \mathbf{r}_v|\,dA$$

S is $\mathbf{r}(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle$

u, v are in D

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mass/area

ex say f is $\rho(x,y,z)$ density of a sheet @ (x,y,z)

2D sheet bent into a shape S

parametrically defined
$\mathbf{r}(u,v) = \langle x,y,z\rangle$
over $u,v \in D$

Mass?

$$M = \iint_S \rho\,dS$$

so

$M = \iint_S \rho \, dS$

$\bar{x} = \frac{1}{M}\iint_S x\rho \, dS$

$\bar{y} = \frac{1}{M}\iint_S y\rho \, dS$

$\bar{z} = \frac{1}{M}\iint_S z\rho \, dS$

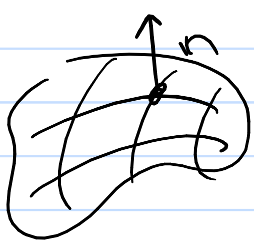
for each .. $\boxed{\text{know}}$

$\iint_S \rho \, dS$

$= \boxed{\iint_D \rho(\mathbf{r}(u,v)) \, |\mathbf{r}_u \times \mathbf{r}_v| \, dA}$

$\iint_S f \, dS$

surface integral of interest..



$\iint_S (\mathbf{F}\cdot\mathbf{n}) \, dS = \iint_S \mathbf{F}\cdot d\mathbf{S}$

called flux of $\mathbf{F}$ across oriented surface S

do it?

$= \boxed{\iint_D \boxed{\mathbf{F}\cdot(\mathbf{r}_u \times \mathbf{r}_v)} \, dA}$

Applications:

① Electric Fields (call $\mathbb{F} \rightsquigarrow \mathbb{E}$)

$$\iint_S \mathbb{F} \cdot d\mathbf{S} \rightsquigarrow \iint_S \mathbb{E} \cdot d\mathbf{S}$$ = electric flux of $\mathbb{E}$ through surface S.

how? $$\iint_D \mathbb{E} \cdot (\mathbf{r}_u \times \mathbf{r}_v)\, dA$$

$\mathbf{r}$ is from S's parametric rep. $\mathbf{r}(u,v) = \langle x, y, z \rangle$

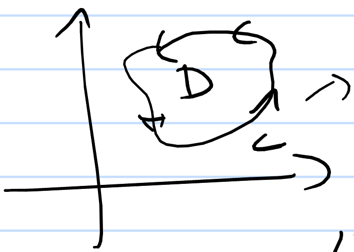
② Heat $$\mathbb{H} = -K \nabla T$$ Temp @ (x,y,z)

$$\iint_S \mathbb{F} \cdot d\mathbf{S} \rightsquigarrow -K \iint_S (\nabla T) \cdot d\mathbf{S}$$

how? $$= -K \iint_D (\nabla T) \cdot (\mathbf{r}_u \times \mathbf{r}_v)\, dA$$

16.8 Stokes

16.5 for Green's Th^{m}



2D

$$\oint_C \mathbb{F}\cdot d\mathbf{r} = \oint_C \langle P, Q\rangle \cdot \langle dx, dy\rangle$$

$\mathbb{F} = \langle P, Q\rangle$

$$= \oint_C P\,dx + Q\,dy$$

$$= \iint_D (Q_x - P_y)\, dA$$

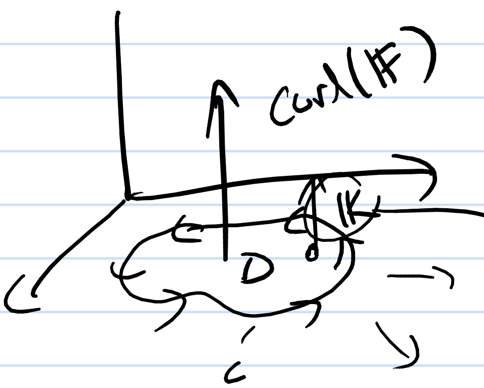
Green's

3D Version

let $\mathbb{F} = \langle P, Q, 0\rangle$

D is still in xy plane

$$\text{curl}(\mathbb{F}) = \langle 0, 0, Q_x - P_y\rangle$$



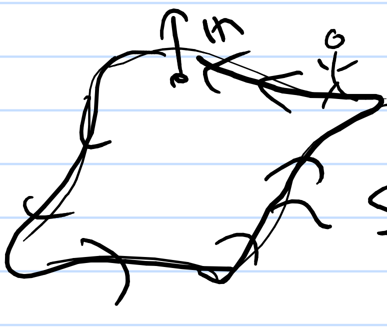
$\mathbb{K} = \langle 0, 0, 1\rangle$ is the normal to D

$\mathbb{F} = \langle P, Q, 0\rangle$

Green's in vector form $\boxed{\oint_C \mathbb{F}\cdot d\mathbf{r} =} \iint_D \text{curl}(\mathbb{F})\cdot\mathbb{K}\, dA$

in general

S is an orientated surface



S's boundry .. call it C

go around C in pos. direction

Stokes S is an oriented piecewise smooth surface with a piecewise smooth boundry C with positive orientation

$\mathbf{F} = \langle P, Q, R \rangle$ is a vector field with cont. partials in an open region containing S.

S with boundry C

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \oint_C \mathbf{F}\cdot\mathbf{T}\, ds$$

$\mathbf{F}$

$$= \iint_S \mathrm{curl}(\mathbf{F})\cdot d\mathbf{S} = \iint_S \mathrm{curl}(\mathbf{F})\cdot \mathbf{n}\, dS$$

"Normal" use

$$\oint_{\partial S} \mathbf{F}\cdot d\mathbf{r} = \iint_S \mathrm{curl}(\mathbf{F})\cdot d\mathbf{S}$$

Surface S has $\mathbf{r}(u,v) = \langle x, y, z \rangle$ over $u,v \in D$

todo $\iint_D \mathrm{curl}(\mathbf{F})\cdot(\mathbf{r}_u \times \mathbf{r}_v)\, dA$