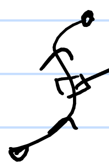
Math 3414

Line Integral



$f(\text{point})$ in ds

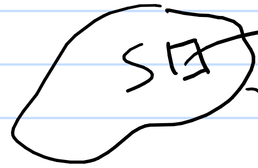
C is $r(t)$ $\quad a \le t \le b$

$$\int_C f \, ds$$

Calculate:

$$\int_C f \, ds = \int_a^b f(r(t)) \, \underline{|r'(t)|} \, dt$$

Surface Integral



$f(\text{point})$ in dS

∂S or C is S's boundry

S is $r(u,v)$ $\quad u,v \in D$

∂S is $r(t)$

$$\iint_S f \, dS$$

Calculate

$$\iint_S f \, dS = \iint_D f(r(u,v)) \, |r_u \times r_v| \, dA$$

$$\int_C xy \, ds \qquad f(x,y) = xy$$

C is $r(t) = \langle t^2, \sin t \rangle$

$0 \le t \le 2\pi$

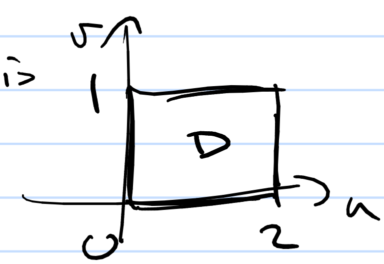
$$= \int_0^{2\pi} t^2 \sin t \sqrt{(2t)^2 + \cos^2 t} \, dt$$

$$= ??$$

$$\iint_S xy + z \, dS \qquad f = xy + z$$

S is $r(u,v) = \langle \underset{x}{\cos u}, \underset{y}{\sin v}, \underset{z}{u+v} \rangle$

D is



$$= \iint_D \left[\cos u \sin v + u + v \right] |r_u \times r_v| \, dA$$

$r_u = \langle -\sin u, 0, 1 \rangle$

$r_v = \langle 0, \cos v, 1 \rangle$

So $r_u = \langle -\sin u, 0, 1 \rangle$

$r_v = \langle 0, \cos v, 1 \rangle$

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ -\sin u & 0 & 1 \\ 0 & \cos v & 1 \end{vmatrix} = \langle -\cos v, \sin u, -\sin u \cos v \rangle$$

$$|r_u \times r_v| = \sqrt{\cos^2 v + \sin^2 u + \sin^2 u \cos^2 v}$$

gives $$\iint_D [\cos u \sin v + u + v] \, |r_u \times r_v| \, dA$$

$$= \int_0^2 \int_0^1 [(\cos u) \sin v + u + v] \sqrt{\cos^2 v + \sin^2 u + \sin^2 u \cos^2 v} \, dv \, du$$

$= ???$

Applied $\int_C f \, ds$

"work" $\int_C (F \cdot T) \, ds$

unit tangent to C

$+ F$

Notation: $\int_C F \cdot dr = \int_a^b F(r(t)) \cdot r'(t) \, dt$

calculate?

Two Special Cases

$\mathbb{F}$

(1) $$\int_C \underbrace{(\nabla f)}_{\mathbb{F}} \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$

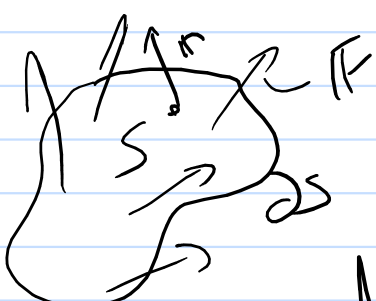
(2) $$\oint_C \mathbb{F} \cdot d\mathbf{r} = \iint_D Q_x - P_y \, dA$$

$\mathbb{F} = \langle P, Q \rangle$

$$= \iint_D \operatorname{curl}(\mathbb{F}) \cdot \mathbf{k} \, dA$$

$\mathbb{F} = \langle P, Q, 0 \rangle$

Applied for $\iint_S f \, dS$



"Flux" $\iint_S (\mathbb{F} \cdot \mathbf{n}) \, dS$

Notation: $\iint_S \mathbb{F} \cdot d\mathbf{S}$

$$\underset{\text{calculate}}{=} \iint_D \mathbb{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) \, dA$$

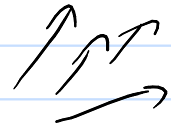
Special Cases

$\boxed{\text{Stokes th}^n}$

$$\iint_S \operatorname{curl}(\mathbb{F}) \cdot d\mathbf{S} = \oint_C \mathbb{F} \cdot d\mathbf{r}$$

$\mathbb{F} = \langle P, Q, R \rangle$

$\xrightarrow{\text{gives}}$ $\operatorname{curl}(\mathbb{F}) = \nabla \times \mathbb{F}$

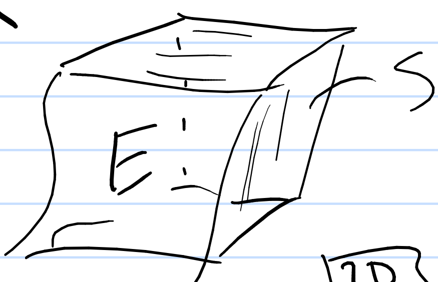


$\boxed{16.9}$ $\boxed{\text{divergence th}^n}$

$$\iint_S \mathbb{F} \cdot d\mathbf{S} = \iint_S (\mathbb{F} \cdot \mathbf{n})\, dS \quad \text{- "flux"}$$

$*$ S is the boundry of a 3D region E

Special case



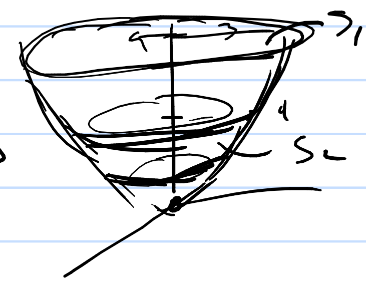
$\boxed{3D}$

$$\iint_S \mathbb{F} \cdot \mathbf{n}\, dS = \iiint_E \operatorname{div}(\mathbb{F})\, dV$$

$\boxed{2D}$

$$\int_C \mathbb{F} \cdot \mathbf{n}\, ds = \iint_D \operatorname{div}(\mathbb{F})\, dA$$

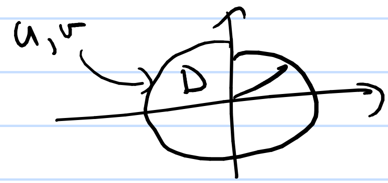
Ex E is region bounded by $\boxed{\begin{matrix} z = 9 \\ z = x^2+y^2 \end{matrix}}$



$\mathbb{F} = \langle y^2 z^3, 2yz, 4z^2 \rangle$

$\iint_S \mathbb{F}\cdot\mathbf{n}\,dS$

S_1 : $\mathbf{r}(u,v) = \langle u, v, 9 \rangle$

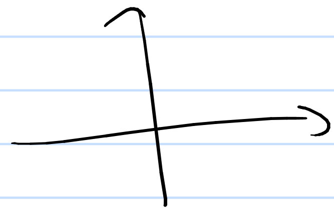


S_2 $\quad z = x^2+y^2$

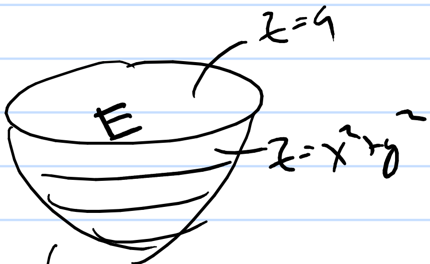
$x = u$

$y = v$

$z = u^2+v^2$



by div. Th^m $\iint_S \mathbb{F}\cdot\mathbf{n}\,dS = \iiint_E \operatorname{div}(\mathbb{F})\,dV$



$\operatorname{div}(\mathbb{F}) = \operatorname{div}(\langle y^2z^3, 2yz, 4z^2 \rangle)$

$= \nabla\cdot\mathbb{F} = 0 + 2z + 8z = 10z$

ch 15 probs

$\iiint_E 10z\,dV$ $\quad$ use cylindrical coord