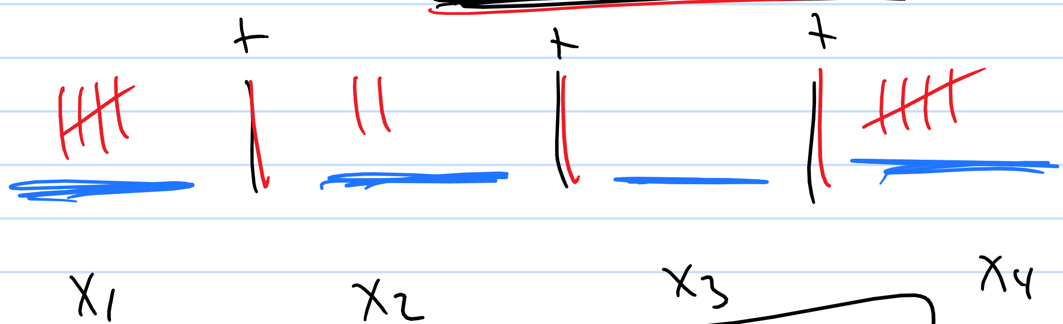
Math 530

Q's 5.4 #13 $x_1 + x_2 + x_3 = 0$ $x_i \ge -5$

example #6 $x_1 + x_2 + x_3 + x_4 = 12$ objects $\boxed{x_i \ge 0}$



$$\boxed{C(12 + 4 - 1, 12)}$$

$$C(15 + 3 - 1, 15)$$

$x_1 + x_2 + x_3 = 15$ $x_i \ge 0$

Substitution $x_1 + x_2 + x_3 = \boxed{0}$ $x_i \ge -5$

let $y_i = x_i + 5$ $y_i = x_i + 5 \ge -5 + 5 = 0$

$y_i - 5 = x_i$ $y_i \ge 0$

$$\boxed{y_1 + y_2 + y_3 = 15 \qquad y_i \ge 0}$$

$$C(15 + 3 - 1, 15)$$

$X_1 + X_2 + X_3 = 0$ $\quad$ $X_i \ge -5$

$\underline{x_1 + x_2 + x_3 = 0}$ $\quad$ $x_1 \overset{+5}{} \ge -5 \overset{+5}{}$ $\quad$ $x_2 \overset{+5}{} \ge -5 \overset{+5}{}$ $\quad$ $x_3 \overset{+5}{} \ge -5 \overset{+5}{}$

$(x_1+5) + (x_2+5) + (x_3+5) = 5 + 5 + 5$

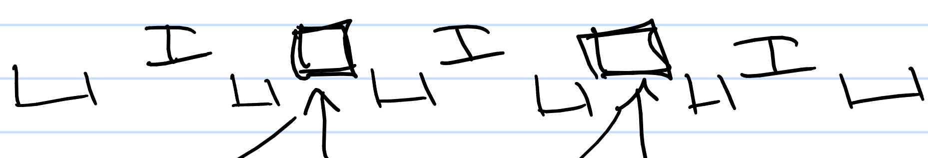
$y_i = x_i + 5$

$y_1 + y_2 + y_3 = 15 \qquad y_i \ge 0$

$C(15+3-1, 15) = \binom{17}{15} = \boxed{\dfrac{17!}{15!\,2!}}$

remember: $\binom{17}{15} = \binom{17}{2}$

#7 VISITING $\qquad \boxed{\binom{6}{3} \cdot 5!}$



I's

$1 \cdot 5 \cdot 4 \; \boxed{\overset{5 \quad\quad 4}{\binom{6}{3} \, 3!}} = \boxed{5 \cdot 4 \cdot P(6,3)}$

(21) 12 tosses 3H + 9T how many have at least 5 tails in a row

$\boxed{TTTTT}$, T, T, T, T, H, H, H

S_1

place S_1 = 8

from next 7 choose 4 for T's

$\underline{\underline{8}} \times \boxed{C(7,4)}$

ans — $4 \times C(4+4-1, 4)$

$\boxed{TTTTT}$ T T T T H H H

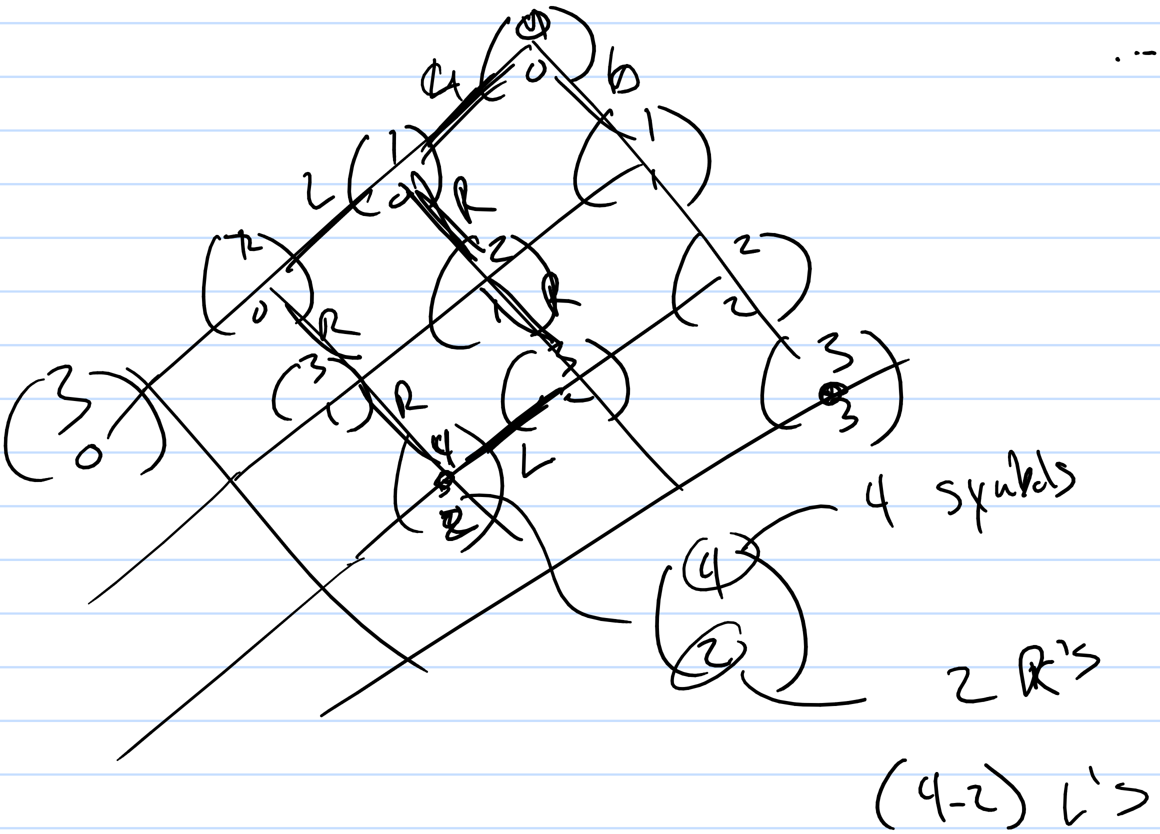
T $\boxed{}$ T T T

T, T, $\boxed{}$, T, T,

at least 5   exactly 5 and exactly 6 or exactly

for Next time

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \dots + \boxed{\binom{n}{r}a^{n-r}b^r} + \dots + \binom{n}{n}a^0 b^n$$



$$(a+b)^n =$$

$$\underbrace{\{a,b\}\{a,b\}\dots\{a,b\}}_{n\text{-times}} \rightsquigarrow \boxed{\binom{n}{r}}\, a^{n-r}b^r$$

ways to select r objects of a certain procedure.

consider $\underline{(a+b)^n} = \underline{\binom{n}{0}} a^n b^0 + \cdots + \underline{\binom{n}{r}} a^{n-r} b^r + \cdots + \binom{n}{n} a^0 b^n$

$$(1+x)^n = \alpha_0 + \alpha_1 x^1 + \alpha_2 x^2 + \alpha_3 x^3 + \cdots + \boxed{\alpha_r} x^r + \cdots + \alpha_n x^n$$

for this $\longrightarrow$ $\alpha_r = \binom{n}{r}$ # of ways to select r objects from n sets of $\{1 \text{ or } x\}$

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generalize this (generating function)

$$g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + \cdots + a_r x^r + \cdots + a_n x^n$$

$$g(x) = a_0 + a_1 x + \cdots + a_r x^r + \cdots \quad \text{(series)}$$

a_r = # of ways to select r objects of a certain procedure

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ex $(1+x)^3 = (1+x)(1+x)(1+x)$

$$= 1\cdot1\cdot1 + \underbrace{1\cdot1\cdot x + 1\cdot x\cdot 1 + x\cdot1\cdot1}_{3x} + \cdots$$

$\begin{Bmatrix}1\\x\end{Bmatrix}\begin{Bmatrix}1\\x\end{Bmatrix}\begin{Bmatrix}1\\x\end{Bmatrix}$: 111, 11x, 1xx, xxx
1x1, x1x
x11, xx1

$(1+x) \sim (x^0 + x^1)$

$\begin{Bmatrix} 1 \\ x \end{Bmatrix}$ $\begin{Bmatrix} x^0 \\ x^1 \end{Bmatrix}$

generalize: x's $\begin{Bmatrix} x^0 \\ x^1 \\ x^2 \\ \vdots \\ x^n \end{Bmatrix}$

$\rightsquigarrow$

$x^{e_1} \quad 0 \le e_1 \le n$

so $(1+x)^n$ is really $(x^0 + x^1)^n$

turns into $\overbrace{\begin{Bmatrix} x^0 \\ x^1 \end{Bmatrix} \begin{Bmatrix} x^0 \\ x^1 \end{Bmatrix} \cdots \begin{Bmatrix} x^0 \\ x^1 \end{Bmatrix}}^{n}$ ex $x^0 x^1 x^0 = 1 \cdot x \cdot 1$

$x^{e_1} \quad x^{e_2} \quad \cdots \quad x^{e_n} \rightsquigarrow x^{e_1 + e_2 + \cdots + e_n}$

want $\underline{r}$

$e_1 + e_2 + \cdots + e_n = r \quad 0 \le e_i \le 1$

ex generating functions $\rightleftarrows$ $\boxed{\text{diophantine eqn}}$

counting prob.

generating function (products of polynomials)

$$(x^0 + x^1 + \cdots + x^k)$$

each polynomial ≡ a type of object

in a polynomial $\begin{Bmatrix} x^0 \\ x^1 \\ \vdots \\ x^k \end{Bmatrix}$ from 0 to k of a type.

x^{e_i} type i

ex

$$(1+x+x^2)^4$$

$$\begin{Bmatrix} x^0 \\ x^1 \\ x^2 \end{Bmatrix}\begin{Bmatrix} x^0 \\ x^1 \\ x^2 \end{Bmatrix}\begin{Bmatrix} x^0 \\ x^1 \\ x^2 \end{Bmatrix}\begin{Bmatrix} x^0 \\ x^1 \\ x^2 \end{Bmatrix}$$

$$x^{e_1} \cdot x^{e_2} \cdot x^{e_3} \cdot x^{e_4}$$

$$\boxed{e_1 + e_2 + e_3 + e_4 = r \quad 0 \le e_i \le 2}$$

generating function for

5 toy trucks, 6 toy tanks, 4 toy planes

select r toys

$$e_1 + e_2 + e_3 = r$$

$$0 \le e_1 \le 5, \quad 0 \le e_2 \le 6, \quad 0 \le e_3 \le 4$$

$$g(x) = \underline{(x^0 + x^1 + x^2 + x^3 + x^4 + x^5)}(x^0 + x^1 + \cdots + x^6)(x^0 + x^1 + \cdots + x^4)$$

$$= a_0 x^0 + a_1 x^1 + a_2 x^2 + \cdots + a_{15} x^{15}$$