Math 530

Q's

5.3 #7

17 people ; D, R, I

choose 17 from 3 types $C(17+3-1, 17)$

$$\binom{17+3-1}{17} = \binom{19}{17} = \frac{19!}{17!\,2!}$$

a) d | r | i

P P P P P P

$d+r+i=17$

$\boxed{\begin{array}{c} d+r+i = \boxed{11} \\ 0 \le d,r,i \end{array}}$

11

17

6

$$\boxed{\binom{11+3-1}{11}} = \binom{13}{11} = \frac{13!}{11!\,2!}$$

choose 6 people → (put the 2 each on committee) (choose 11 more from our 3 types)

1 · $\boxed{\binom{11+3-1}{11}}$

b) | no $\boxed{\text{majority}}$ allowed | 17 people

$$d+r+i = 17 \qquad 2 \le d,r,i \le 8$$

1) | all ≥ 2 | $-$ | majority | $=$ | no majority |

$\boxed{4} - 17$

13

PPPP
PPPP
P | P P | PP

$$\binom{11+3-1}{11} - 3\binom{4+3-1}{4}$$

$\boxed{\text{ch 6}}$ generating functions

$$g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + \cdots + \boxed{a_r} x^r + \cdots$$

a_r: solution to number of ways to select r objects of a certain procedure

$\boxed{6.1}$ $g(x)$ is a product of polynomials

each polynomial $\rightsquigarrow$ $\left(\begin{smallmatrix}1\\ \text{or}\\ 0\end{smallmatrix} x^0 + \begin{smallmatrix}1\\ \text{or}\\ 0\end{smallmatrix} x^1 + \begin{smallmatrix}1\\ \text{or}\\ 0\end{smallmatrix} x^2 + \cdots + \begin{smallmatrix}1\\ \text{or}\\ 0\end{smallmatrix} x^k \right)$

is a $\boxed{\text{type}}$

for the type:
x^0 is 0 of type
$\vdots$
x^k is k of type

0 to k of type

x^e — e is how many of our type

ex (above)

back to $\overbrace{d, r, i}^{3 \text{ types}}$

stack prob. for i majority (need from 9 to ~~17~~ 13)

$$\left.\begin{array}{l} i \rightsquigarrow (x^9 + x^{10} + x^{11} + \cdots + x^{13}) \\ d \rightsquigarrow (x^2 + \cdots + x^6) \\ r \rightsquigarrow (x^2 + \cdots + x^6) \end{array}\right] e_i + e_d + e_r =$$

$$g(x) = (x^9 + x^{10} + \cdots + x^{13})(x^2 + \cdots + x^6)^2$$

Importance of zero (position matters)

ex of some $g(x) = (1 + x^5 + x^{10})^2(1 + x^2 + x^4)$

$$= (1 + 0x + 0x^2 + 0x^3 + 0x^4 + x^5 + 0x^6$$

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ex 8 $\underbrace{\text{flavors}}_{\text{types}}$ of gum each are in packs of 5

type $\rightsquigarrow$ $\boxed{\text{poly}}$ $(x^0 + x^5 + x^{10} + \cdots)$

$$\boxed{g(x) = (1 + x^5 + x^{10} + \cdots)^8}$$

$\boxed{6.2}$ How to go from (prod of poly) = $g(x)$

Have ① $(1+x)^n = \binom{n}{0}x^0 + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \cdots$

$$\boxed{\begin{aligned} (x^0+x^1)^3 &= (x^0+x^1)(x^0+x^1)(x^0+x^1) \\ (1+x)^3 &= 1x^0 + 3x^1 + 3x^2 + 1x^3 \end{aligned}}$$

$\boxed{\text{have}}$

① $(1+x)^n = \binom{n}{0}x^0 + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$

② $(1-x^m)^n = 1 - \binom{n}{1}x^m + \binom{n}{2}x^{2m} + \cdots + (-1)^k\binom{n}{k}x^{km}$

③ $\dfrac{1-x^{m+1}}{1-x} = 1 + x + x^2 + \cdots + x^m$

④ $\dfrac{1}{1-x} = 1 + x + x^2 + \cdots$

⑤ $\dfrac{1}{(1-x)^n} = \left(\dfrac{1}{1-x}\right)^n = 1 + \binom{1+n-1}{1}x + \binom{2+n-1}{2}x^2$

$+ \cdots + \binom{r+n-1}{r}x^r + \cdots$

(poly)(poly) ?

$$(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots)(b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \cdots)$$

$$= (a_0 b_0) + (a_0 b_1 + a_1 b_0)x + (a_0 b_2 + a_1 b_1 + a_2 b_0)x^2 + \cdots$$

$$\left(a_0 b_5 + a_1 b_4 + a_2 b_3 + a_3 b_2 + a_4 b_1 + a_5 b_0\right) x^5$$

$$s(x) = (x^2 + x^3 + x^4)(x^2 + x^3 + x^4 + x^5)$$

$$= \boxed{x^4}(1 + x + x^2)(1 + x + x^2 + x^3)$$

select $6 = a_6$ & $a_6 x^6$

4 need x^2 of

$$(1\cdot 1 + 1\cdot 1 + 1\cdot 1)x^2 = 3x^2$$

So $x^4(\sim + 3x^2 + \cdots)$

$$\sim + \boxed{3}x^6 + \cdots$$

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