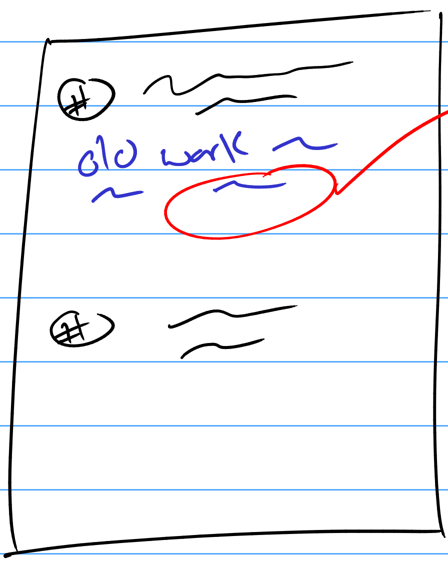
Math 530

Exam 1

Grading ... what I want?

take your exam..



wrong at this point.. b/c (explain)

ex $= \frac{7!}{2!\,5!} = \frac{7!}{2!\,4!} =$?

copy error

$\boxed{-1}$

ex $C(7,2)$

No.. order matters! $P(7,2)$

-6 ← large part?

-3 ← smaller part?

etc

Math 530 ... Exam 1

0) Exam Start Time:

0) Name: Exam Key

0) MyWSUid:

0) Sign below to state that you have read and understand the WSU policy on Student Academic Integrity

https://www.wichita.edu/about/policy/ch_02/ch2_17.php

0) Please explain all your answers in the exam.

1) Your kitchen has seven styles of hot sauce that you have recipes for. For each style of hot sauce, there are 4 different peppers, 3 different citrus fruits, and 12 different spices you could use. How many different types of hot sauce could you make?

$\boxed{7\cdot 4\cdot 3\cdot 12}$

recipes | peppers | fruits | spices

$7\cdot\left[4+\binom{4}{2}+\binom{4}{3}+\binom{4}{4}\right]$

$\cdot\,[\text{ etc }]$

$\cdot\,[\text{ etc }]$

2) Do problem 21 of section 5.1 ~~along with parts~~ a and c.

3 letter string w/o repeats using a, b, c, d, e, f

in which e or f (or both) is used?

$|\text{all}| - |\text{no } e \text{ or } f| = \boxed{6\cdot5\cdot4 - 4\cdot3\cdot2}$

$3\times2\times\underbrace{4\times3}_{\text{other } 2}$

place e, place f, other 2

(c) $3\times2\times\underbrace{4\times4}_{\text{other w/ repeat}} - \overbrace{3\times2\times4}^{e,\,f,\text{ one other}}$

place e, place f

minus

3) You have 15 people show up to play basketball. How many ways to choose two teams of 5 people? How many ways to $\boxed{\text{pick}}$ 5 people for the first team and then everyone else is on the second team? Also give one wrong answer for the second question and explain why it is wrong and why someone could do it that way.

(a)
$$\underbrace{\binom{15}{5}}_{\text{choose } 5} \cdot \underbrace{\binom{10}{5}}_{\text{choose next } 5} = \boxed{\frac{15!}{5!\,10!} \cdot \frac{10!}{5!\,5!}} = \boxed{\frac{15!}{5!\,5!\,5!}}$$

of 15 you have

team 1 = 5
team 2 = 5
no chosen = 5

(b) $P(15,5) \cdot \boxed{1}$

$\binom{10}{10}$

$\cancel{10!}$

5pt?

$$= \boxed{\frac{15!}{10!}}$$

below is any combo including 0-starts.

4) How many 7 digit phone numbers have one or more repeated digits?

See example 7 p. 146

Same as $|\text{all}| - |\text{no repeats}|$

just pick 7 digits and order matters.

example 7

$$= 10^7 - \underline{P(10,7)}$$

$$= \boxed{10^7 - \frac{10!}{3!} = 10^7 - 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4}$$

5) How many ways are there to roll a die 10 times? How many ways are there to roll a die 10 times and obtain six 1's, two 2's, one 3, and one 4?

roll 1 , roll 2 , roll 3, .. , roll 10

6   6   6 ... 6

so (a) $\boxed{6^{10}}$

(b) arrangements of 1, 1, 1, 1, 1, 1, 2, 2, 3, 4

$$P(10;6,2,1,1) = \boxed{\frac{10!}{6!\,2!\,1!\,1!}} = \binom{10}{6}\binom{4}{2}\binom{2}{1}\binom{1}{1}$$

6) You have 20 sticks, 15 rocks, and 7 pieces of string in your pocket. How many ways to take 5 things out of your pocket? How many ways to take 10 things out of your pocket?

3 types

(a) take 5 of 3 types $\boxed{\binom{5+3-1}{5}} = \binom{7}{5} = \boxed{\frac{7!}{5!\,2!}}$

(b) take 10 of 3 types .. $\binom{10+3-1}{10} = \binom{12}{10} = \frac{12!}{10!\,2!}$

but only 7 pieces of string so .. remove any selections using 8 pieces of string and other 2 of 3 types ...

$$\frac{12!}{10!\,2!} - \binom{2+3-1}{2} = \frac{12!}{10!\,2!} - \binom{4}{2} = \boxed{\frac{12!}{10!\,2!} - \frac{4!}{2!\,2!}}$$

7) Write problem 5 of section 5.4 as three diophantine equations. One for chocolate doughnuts, one for cinamon doughnuts, and one for powdered sugar doughnuts.

P_i = principal$_i$ amount

Chocolate: $P_1 + P_2 + P_3 + P_4 = 18 \qquad P_i \geq 2$

Cinamon: $P_1 + P_2 + P_3 + P_4 = 12 \qquad P_i \geq 2$

sugar: $P_1 + P_2 + P_3 + P_4 = 14 \qquad P_i \geq 2$

$P_i \geq 0 \quad P_1 + P_2 + \cdots + P_n = r \rightarrow \boxed{\binom{r+n-1}{r}}$

ex 5 p. 217

8) Show that the number of ways to distribute r identical toys to n children with at least one toy for each child is $C(r-1, n-1)$.

Same as give 1 toy to each child and new problem is distribute $r-n$ toys to n children

$$\binom{(r-n)+n-1}{r-n} = \binom{r-1}{r-n} = \frac{(r-1)!}{(r-n)!\,[(r-1)-(r-n)]!}$$

$$= \frac{(r-1)!}{(r-n)!\,(n-1)!} = \boxed{\binom{r-1}{n-1}}$$

$(r-n)+(n-1)$
$= r-1$

9) Prove the binomial theorem by induction. $(1+x)^n = \sum_{i=0}^{n} \binom{n}{i} x^i$

Basis: $\boxed{n=0}$: $(1+x)^0 \stackrel{?}{=} \sum_{i=0}^{0} \binom{0}{i} x^i \to 1 \stackrel{?}{=} 1$ true

Inductive step: assume $(1+x)^k = \sum_{i=0}^{k} \binom{k}{i} x^i = 1 + \binom{k}{1}x + \binom{k}{2}x^2 + \dots$

$$(1+x)^{k+1} = (1+x)(1+x)^k = (1+x)\sum_{i=0}^{k}\binom{k}{i}x^i$$

$$= (1+x)\left[\binom{k}{0} + \binom{k}{1}x + \binom{k}{2}x^2 + \dots + \binom{k}{k}x^k\right]$$

$$= 1 + \left[\binom{k}{0} + \binom{k}{1}\right]x + \left[\binom{k}{1} + \binom{k}{2}\right]x^2 + \dots + 1 \cdot x^{k+1}$$

Pascal's identity $= 1 + \binom{k+1}{1}x + \binom{k+1}{2}x^2 + \dots + \binom{k+1}{k+1}x^{k+1}$ true

10) Do problem 14c of section 5.5.

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$$

let $x = 2$

so $\boxed{(3)^n = \sum_{k=0}^{n} 2^k \binom{n}{k}}$

11) Create a selection word problem for the following expression $(1 + x + x^3)^2(x^2 + x^4)$.

$$\underbrace{(x^0 + x^1 + x^3)}_{\text{type 1}}\underbrace{(x^0 + x^1 + x^3)}_{\text{type 2}}\underbrace{(x^2 + x^4)}_{\text{type 3}}$$

has 0, 1 or 3   has 0, 1 or 3   has 2 or 4

Word prob?

12) Write your solution to homework problem 3 of section 6.2.

$a_7\ x^7$ of $(1 + x^2 + x^4)(1 + x)^n$

$$\to (1 + x^2 + x^4)\left(1 + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n\right)$$

take x^7 of right   take x^5 of right   take x^3 of right

so $\boxed{\binom{n}{7} + \binom{n}{5} + \binom{n}{3}}$

0) What is the time you ended working on the exam and started scanning it?