Math 530

Q's 7.1 (1, 5, 7, 11, 15, 19)
7.2 (1bdf)
7.3 (1, 3ac, 5)

Recurrence relation: $a_n = f(a_{n-1}, a_{n-2}, \dots, a_0)$

Initial values $a_0 = \ ,\ a_1 = \ \dots,\ a_k =$

"solve" find a closed function $a_n = \boxed{a(n)}$

expression involving only n's

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types of $a_n = f(a_{n-1}, a_{n-2}, \dots, a_0)$

#1 linear homogeneous rec. relation of degree k with constant coeff.

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$$

Answer: $a_n = r^n$ for some constant r's

(so find the r's)

$a_0=1 \quad a_1=1 \quad a_2=1 \quad a_3=1 \qquad (a+b)^3 = a^3+3a^2b+3ab^2+b^3$

ex

recurr. rel.

$$a_n = 5a_{n-1} - 6a_{n-2} - 4a_{n-3} + 8a_{n-4}$$

$$r^4 = 5r^3 - 6r^2 - 4r + 8$$

$$r^4 - 5r^3 + 6r^2 + 4r - 8 = 0$$

$$\begin{array}{r|l}
 & r^3 - 6r^2 + 12r - 8 \\
\hline
(r+1) & r^4 - 5r^3 + 6r^2 + 4r - 8 \\
 & r^4 + r^3 \\
\hline
 & -6r^3 + 6r^2 \\
 & -6r^3 - 6r^2 \\
\hline
 & 12r^2 + 4r \\
 & 12r^2 + 12r \\
\hline
 & -8r - 8 \\
 & -8r - 8 \\
\hline
 & 0
\end{array}$$

$$(r+1)(r^3 - 6r^2 + 12r - 8) = 0$$

$$(r+1)(r-2)^3 = 0$$

$$a_n = (A)(-1)^n + (B + Cn + Dn^2)(2)^n$$

$$\left.\begin{array}{l} a_0 = 1 \\ a_1 = 1 \\ a_2 = 1 \\ a_3 = 1 \end{array}\right] \quad \begin{array}{l} 1 = A + B \\ 1 = -A + 2B + 2C + 2D \\ 1 = A + 4B + 8C + 16D \\ 1 = -A + 8B + 24C + 72D \end{array}$$

Lin Alg. **Solve:**

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ -1 & 2 & 2 & 2 & 1 \\ 1 & 4 & 8 & 16 & 1 \\ -1 & 8 & 24 & 72 & 1 \end{array}\right]$$

ex

n feet

2ft red

1ft yellow

1ft blue

$a_n = r^n$

$$a_n = (1)\,a_{n-2} + (2)\,a_{n-1}$$

red — 2ft flag

yellow or blue — 1ft flag

$a_0 = 1$ $\quad a_1 = 2$ $\quad a_2 = 5$ $\qquad 1, 2, 5, 12, 29, \ldots$

$$a_n = 2a_{n-1} + a_{n-2}$$

"Solve"

$$r^2 - 2r - 1 = 0$$

$$r = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$r = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

$$r_1 = 1 + \sqrt{2} \qquad r_2 = 1 - \sqrt{2}$$

$$a_n = (A)(1+\sqrt{2})^n + (B)(1-\sqrt{2})^n \qquad a_0 = 1,\; a_1 = 2$$

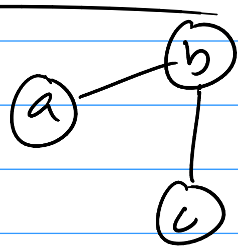
$a_0 = 1$, $a_1 = 2$

$$\begin{cases} 1 = A + B \\ 2 = A(1+\sqrt{2}) + B(1-\sqrt{2}) \end{cases}$$

Solve for A, B

7.2 Divide-and-Conquer Relations

(ex) S is a matrix



$$S = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

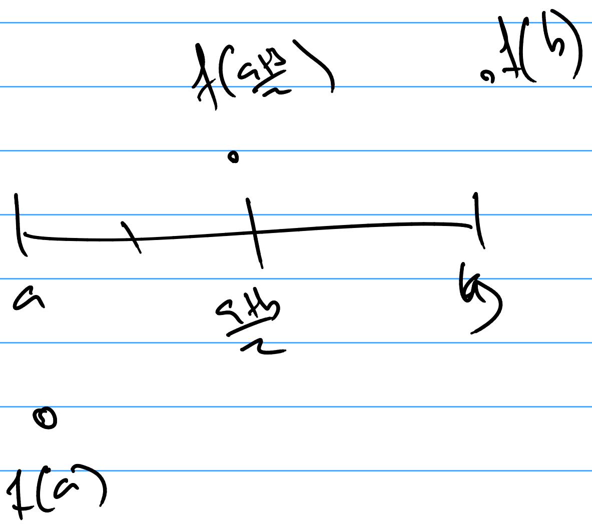
$$S_1 = S$$
$$S_2 = S + S^2 = S_1(I + S_1)$$
$$S_3 = S + S^2 + S^3$$
$$S_4 = S + S^2 + S^3 + S^4 = S_2(I + S_2)$$

$$S_n = S + S^2 + S^3 + \cdots + S^n$$

$$S_1 = S$$
$$S_{2^k} = S_{2^{k-1}}(I + S_{2^{k-1}})$$

$$S_1 = S$$
$$S_2 = S_1(I + S_1)$$
$$S_4 = S_2(I + S_2)$$
$$S_8 = S_4(I + S_4)$$
$$S_{16} = S_8(I + S_8)$$
$$S_{32} = S_{16}(I + S_{16})$$

$$S + S^2 + S^3 + \cdots + S^{32}$$



$a_n = c\, a_{n/2} + f(n)$

steps required for a divide and conquer algorithm to process n-objects

found by initial value

$a_n = a_{n/2} + d \xrightarrow{\text{soln}} a_n = a(n) = d\lceil \log_2 n \rceil + A$

$a_n = 2a_{n/2} + d \longrightarrow a_n = a(n) = An - d$

$a_n = c\, a_{n/2} + dn \longrightarrow a_n = a(n) = A \cdot n^{\log_2 c} + \left(\frac{2d}{2-c}\right)n$

$c > 2$

$a_n = 2a_{n/2} + dn \rightarrow a_n = a(n) = dn(\lceil \log_2 n \rceil + A)$