Math 530

HW 7.5 (examples 1 and 2 with student level details)

Q's

7.1 #14

Under stamp problems

Not #14 (eg) $N = a\$5 + b\$3$

$\$3 = 1\cdot\$3 + 0\$5$
$\$5 = 0\cdot\$3 + 1\$5$
$\$6 = 2\$3 + 0\cdot\$5$
$\$8 = 1\cdot\$3 + 1\cdot\$5$
$\$9 = 3\cdot\$3 + 0\cdot\$5$
$\$10 = 0\cdot\$3 + 2\cdot\$5$
$\$11 = 2\cdot\$3 + 1\cdot\$5$
$\$12$

+1·$3

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14 a) $1, $3, $5

a_n = $1 \cdot a_{n-1}$ + $1 \cdot a_{n-3}$ + $1 \cdot a_{n-5}$

↑ $n — start with $1 — or start w/ $3 — or start w/ $5

$$a_n = a_{n-1} + a_{n-3} + a_{n-5}$$

14 b) $a_n = a_{n-1} + a_{n-3} + a_{n-5} - a_{n-6}$

start w/ 1 — start w/ 3 — start with 5

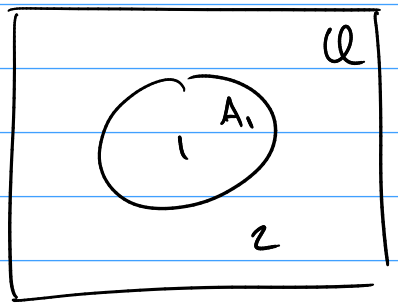
but I don't want 5,1 starts

$1 \cdot a_{n-6}$

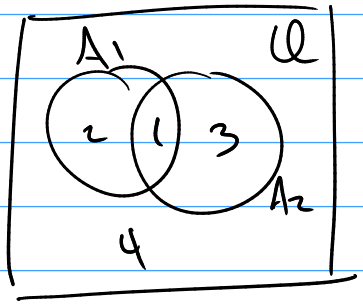
Small n 6

Ch 8 Inclusion-Exclusion Principle(s)

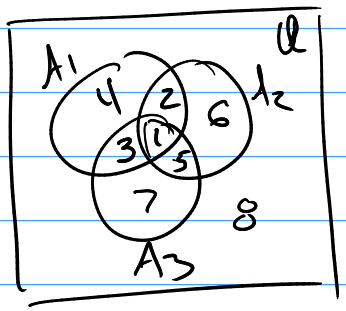
Venn diagram



1 set



2 sets



3 sets

Membership tables

	A_1
row 1	1
row 2	0

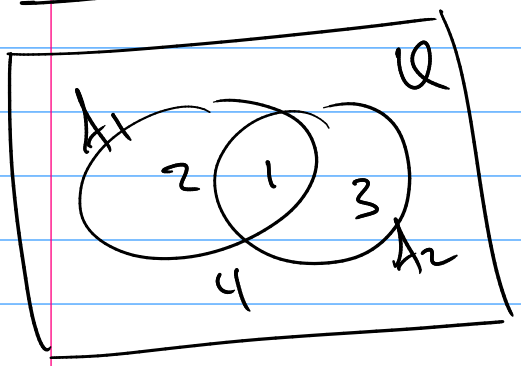
region $2^{|sets|}$

A_1	A_2	row
1	1	1
1	0	2
0	1	3
0	0	4

A_1	A_2	A_3	rows
1	1	1	1
1	1	0	2
1	0	1	3
1	0	0	4
0	1	1	5
0	1	0	6
0	0	1	7
0	0	0	8

Some Set equalities

(1)

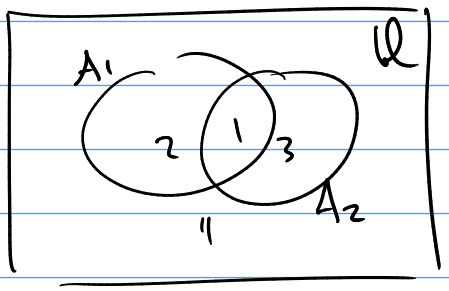


A_1	A_2	
1	1	$\leftarrow A_1 \cap A_2 = A_1 A_2$
1	0	$\leftarrow A_1 - (A_1 A_2)$
0	1	$\leftarrow A_2 - (A_1 A_2)$
0	0	$\leftarrow \overline{(A_1 \cup A_2)} = \overline{A_1}\,\overline{A_2}$

(2) disjoint Notn: $|S| = n(S) =$ # of elements in S

(4) $|U| = |\text{region 1}| + |\text{region 2}| + |\text{region 3}| + |\text{region 4}|$

Concept of inclusion / exclusion



regions 1,2 ↓ regions 1,3 ↓

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 A_2|$$

(↑ region 1)

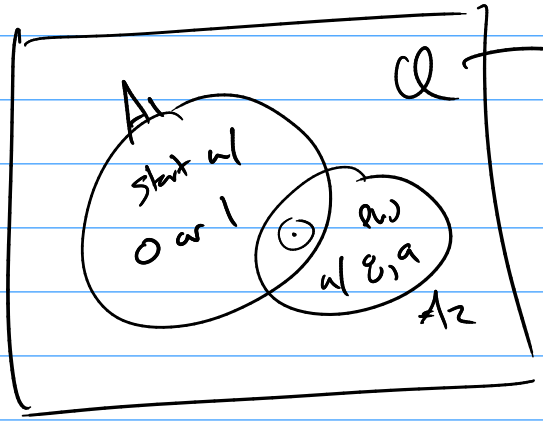
for $|\mathcal{U}| = n$

$$|\bar{A}_1 \bar{A}_2| = |\overline{A_1 \cup A_2}| = |\mathcal{U}| - |A_1 \cup A_2|$$

$$= |\mathcal{U}| - |A_1| - |A_2| + |A_1 A_2|$$

($|\mathcal{U}|$: regions 1,2,3,4; $|A_1|$: region 1,2; $|A_2|$: regions 1,3)

Example 2



arrange 0,1,2,3,4,5,6,7,8,9

$|\mathcal{U}| = 10!_0$

how many have 1st as >1 and last as <8

$$|\bar{A}_1 \bar{A}_2| = |\mathcal{U}| - |A_1| - |A_2| + |A_1 A_2|$$

$$= 10!_0 - 2 \cdot 9!_0 - 2 \cdot 9!_0 + 2 \cdot 2 \cdot 8!_0$$

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 A_2|$$

(↑ start wl 0,1 or no wl 8,9)

$$= 2 \cdot 9!_0 + 2 \cdot 9!_0 - 2 \cdot 2 \cdot 8!_0$$

$$= (2 \cdot 9 + 2 \cdot 9 - 4) 8!_0 \equiv 2(9+9-2)8!_0$$

$$= (9 \cdot 8 \cdot 8!_0) \qquad = 9(9-1)8!_0$$

Sets $A_1, A_2, \ldots, A_n$

Ex

Notation: $|A_1| + |A_2| + \cdots + |A_n| = S_1$

choose 2 sets $\longrightarrow$ $|A_1 A_2| + |A_1 A_3| + \cdots + |A_4 A_6| + \cdots + |A_{n-1} A_n| = S_2$

choose 3 sets $\longrightarrow$ $|A_1 A_2 A_3| + \cdots + |A_{n-2} A_{n-1} A_n| = S_3$

$\vdots$

choose n $\quad |A_1 A_2 \cdots A_n| = S_n$

Inclusion / Exclusion

Corollary $|A_1 \cup A_2 \cup \cdots \cup A_n| = S_1 - S_2 + S_3 - S_4 + \cdots + (-1)^{n-1} S_n$

th^{m} $|\bar{A}_1 \bar{A}_2 \cdots \bar{A}_n| = |U| - S_1 + S_2 - S_3 + \cdots + (-1)^n S_n$