Math 530

Q3 7.3 (#1) 500 invested @ 8%/yr

$$a_n = a_{n-1} + 0.08a_{n-1} = 1.08a_{n-1}$$

$$a_n = 1.08a_{n-1}$$ Solve?

Iterative
$a_0 = 500$
$a_1 = 1.08 \cdot 500$
$a_2 = 1.08 \cdot 1.08 \cdot 500$
$a_2 = 1.08^2 500$
⋮
$a_n = 500 \cdot 1.08^n$

$$r^1 - 1.08 = 0$$
$$r = 1.08$$

Soln: $a_n = A(r)^n$

$$a_n = A(1.08)^n$$

Basis / Initial Value

$a_0 = 500 \rightarrow 500 = A(1.08)^0 = A$

$$a_n = 500(1.08)^n$$

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(1) $|A_1 \cup A_2 \cup \cdots \cup A_n| = S_1 - S_2 + S_3 - S_4 + \cdots + (-1)^{n-1}S_n$

(2) $|\bar{A}_1 \bar{A}_2 \bar{A}_3 \cdots \bar{A}_n| = N - S_1 + S_2 - S_3 + \cdots + (-1)^n S_n$

Where:
$S_1 = |A_1| + |A_2| + \cdots + |A_n|$
$S_2 = |A_1A_2| + |A_1A_3| + \cdots + |A_{n-1}A_n|$
$S_3 = |A_1A_2A_3| + |A_1A_2A_4| + \cdots + |A_{n-2}A_{n-1}A_n|$
⋮
$S_n = |A_1A_2 \cdots A_n|$

Variation of (2)

$$|B_1 B_2 B_3 \cdots B_n| = |\overline{\bar{B}_1}\ \overline{\bar{B}_2}\ \overline{\bar{B}_3} \cdots \overline{\bar{B}_n}|$$
$$= N - S_1 + S_2 - S_3 + \cdots + (-1)^n S_n$$

$$S_1 = |\bar{B}_1| + |\bar{B}_2| + \cdots + |\bar{B}_n|$$
$$S_2 = |\bar{B}_1 \bar{B}_2| + |\bar{B}_1 \bar{B}_3| + \cdots$$
$$\vdots$$

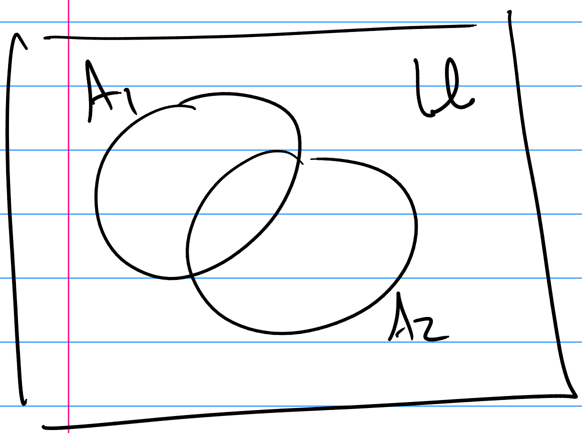
HW

8.1

(1) 8 letter words $\quad l_1 l_2 l_3 \cdots l_8$

$$|\text{8 letter words}| = 26 \cdot 26 \cdot \cdots 26 = 26^8$$

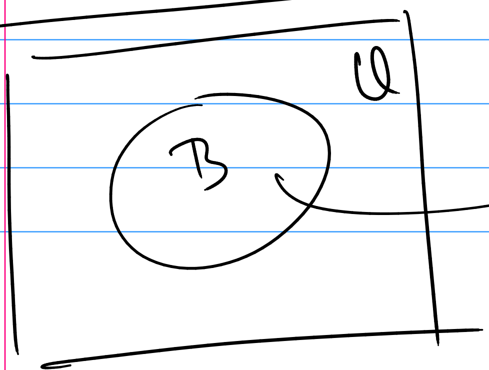
probability of 8 letter words & the 26 letters (repeats ok) [illegible] start or end with a vowel.



U ≡ all 8 letter words
A_1 ≡ start with vowel
A_2 ≡ end with vowel

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 A_2|$$

$$|A_1| = 5 \cdot 26^7 \qquad \text{ans} = 5 \cdot 26^7 + 5 \cdot 26^7 - 25 \cdot 26^6$$
$$|A_2| = 5 \cdot 26^7$$
$$|A_1 A_2| = 5 \cdot 5 \cdot 26^6$$



$$|\bar{B}| = N - |B|$$

does not begin with a vowel and does not end with a vowel

$\overline{B} \equiv$ begins or ends w/ vowel $|\overline{B}|$

$|\overline{B}| = 26^8 - 21 \cdot 21 \cdot 26^6$

$= 26^8 - 21^2 26^6$

$5 \cdot 26^7 + 5 \cdot 26^7 - 5^2 \cdot 26^6 \overset{?}{=} 26^8 - 21^2 26^6$

$5 \cdot 26^6 [26 + 26 - 5] \qquad 26^6 [26^2 - 21^2]$

(ex) 15 digits (digit : {0, 1, 2, .., 9})

$d_1 d_2 d_3 \cdots d_{15}$

How many are there in which digits 1, 2, 3 all appear?

$|\text{has } 1 \wedge \text{has } 2 \wedge \text{has } 3|$

$= |\overline{\neg 1} \wedge \overline{\neg 2} \wedge \overline{\neg 3}|$

$= \text{all} - S_1 + S_2 - S_3$

$= 10^{15} - |\neg 1| - |\neg 2| - |\neg 3|$ (each 9^{15})

$+ |\neg 1 \wedge \neg 2| + |\neg 1 \wedge \neg 3| + |\neg 2 \wedge \neg 3|$ (each 8^{15})

$- |\neg 1 \wedge \neg 2 \wedge \neg 3|$ (7^{15})

$$\text{ans} = \boxed{10^{15} - 3\cdot 9^{15} + 3\cdot 8^{15} - 7^{15}}$$

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15 digits, have a 1

$$|\text{have a } 1| = \left|\begin{matrix}\text{exactly}\\ 1-1\end{matrix}\right| + \left|\begin{matrix}\text{exactly}\\ 2-1\text{'s}\end{matrix}\right| + \dots + \left|\begin{matrix}\text{exactly}\\ 5-1\text{'s}\end{matrix}\right|$$

$$|\text{all}| = |0-1\text{'s}| + \left|\begin{matrix}\text{exactly}\\ 1-1\end{matrix}\right| + \dots + \left|\begin{matrix}\text{exactly}\\ 5-1\text{'s}\end{matrix}\right|$$

$$10^{15} - 9^{15}$$