Math 530

8.1 (1, 3, 5, 11, 17, 23)
8.2 (1, 3, 7, 13, 17)

7.5 (rewrite example 1 , example 2)

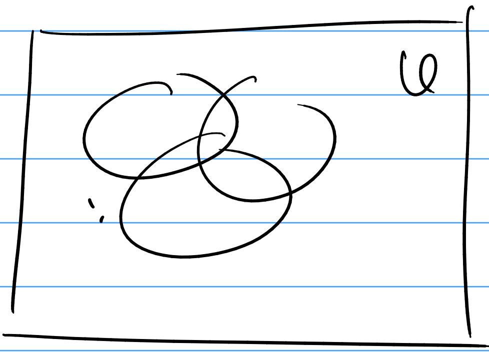
p. 306 — p. 310

$S_1 = |A_1| + |A_2| + \dots + |A_n|$

$S_2 = |A_1 A_2| + \dots$

$S_3 = |A_1 A_2 A_3| + \dots$

$S_n = |A_1 A_2 \dots A_n|$



$|U|$ = all possible in universe

$$|\overline{A_1}\,\overline{A_2} \dots \overline{A_3}| = |U| - S_1 + S_2 - \dots$$

$$|A_1 \cup A_2 \cup \dots A_n| = S_1 - S_2 + S_3 -$$

A_i? $\overline{A_i}$?

→ $A_i = \{ e \mid$ logical predicate on $e \}$

→ $\overline{A_i} = \{ e \mid \neg$ logical predicate on $e \}$

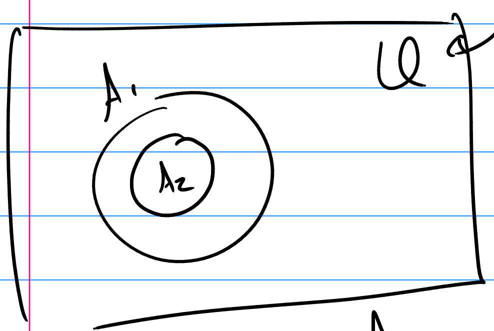
8.2

(3) 9-card hand

4 suits of {13}

{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A}

total 52 cards



$$|U| = \binom{52}{9} = \frac{52!}{9!\,43!}$$

A_2 = has two one of a kinds

A_1 = at least one 4 of a kind

$$|A_1| = |all| - |\text{no 4 of a kind @ all}|$$

$$|A_1| = (13)^{\text{card}} \cdot \binom{52-4}{5} - \binom{13}{2}\binom{52-8}{1}$$

by construction get a 4 of a kind {4 cards}

$$\boxed{\text{ans}} = \frac{\binom{13}{1}\binom{52-4}{5} - \binom{13}{2}\binom{52-8}{1}}{\binom{52}{9}}$$

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ex **Birthday Paradox**

prob of N people having some of them have same birthday?

$S = \{1, 2, \ldots, 365\}$

String of length n made of S

$\boxed{\text{ex}}$ 1, 360, 4 string of length 3

p_1, p_2, p_3, p_4, p_5

Pairs: $4+3+2+1$

$5 \rightarrow \frac{4(5)}{2}$

$10 \rightarrow \frac{9(10)}{2} = 45$

$20 \rightarrow \frac{19(20)}{2} = 190$

???

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length n

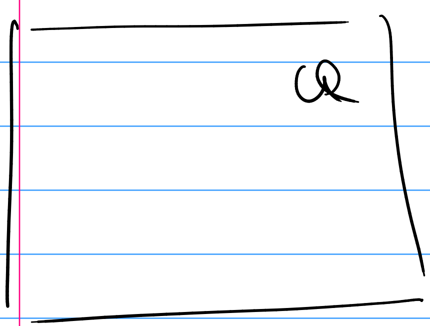
$$|U| = 365^n$$

$$|\text{no common bday}| = P(365, n)$$

$$\text{Prob}(\text{a bd}) = \frac{365^n - \frac{365!}{(365-n)!}}{365^n} = 1 - \frac{\frac{365!}{(365-n)!}}{365^n}$$

$a, a, a, b, b, b, c, c, c$ (1, 2, 3, 4, 5, 6, 7)

all arrangements w/o 3 consecutive letters being the same



$$|U| = \frac{9!}{3!\,3!\,3!}$$

$$|\overline{A_a} \wedge \overline{A_b} \wedge \overline{A_c}|$$

$\overline{A_a} \equiv \neg(3 \text{ consect. } a\text{'s})$

$\overline{A_b} \equiv \neg(3 \text{ consect. } b\text{'s})$

$\overline{A_c} = \neg(3 \text{ consect } c\text{'s})$

$$|\bar{A}_a \cap \bar{A}_b \cap \bar{A}_c| = |U| - S_1 + S_2 - S_3$$

consecu. a's

A_a

$\boxed{aaa}, b, b, b, c, c, c$

$$S_1 = |A_a| + |A_b| + |A_c|$$

$$= \frac{7!}{3!\,3!} + \frac{7!}{3!\,3!} + \frac{7!}{3!\,3!} = \boxed{3 \cdot \frac{7!}{3!\,3!}}$$

A_aA_b

$\boxed{aaa}, \boxed{bbb}, c, c, c$

$$S_2 = |A_aA_b| + |A_aA_c| + |A_bA_c|$$

$$= \frac{5!}{3!} + \frac{5!}{3!} + \frac{5!}{3!} = \boxed{3 \cdot \frac{5!}{3!}}$$

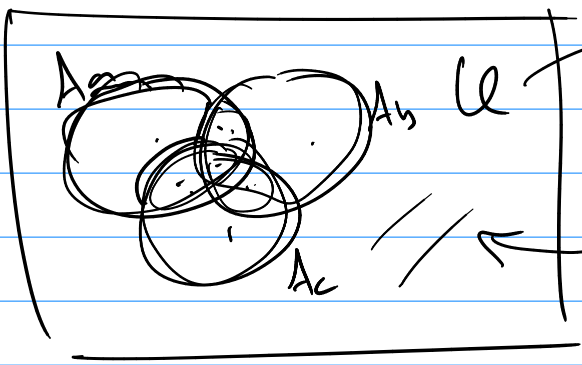
$A_aA_bA_c$

$\boxed{aaa}, \boxed{bbb}, \boxed{ccc}$

$$S_3 = |A_aA_bA_c|$$

$$= 3!$$

$$\boxed{\text{Ans}} = \frac{9!}{3!\,3!\,3!} - 3\,\frac{7!}{3!\,3!} + 3\,\frac{5!}{3!} - 3!$$



any arrange of $a, a, a, b, b, b, c, c, c$

$|\bar{A}_a \bar{A}_b \bar{A}_c|$

3

$$\frac{\binom{13}{1}\binom{52-4}{5} - \binom{13}{2}\binom{52-8}{1}}{\binom{52}{9}}$$

number w/ at least one 4 of a kind

$$13\binom{48}{5} - \frac{13!}{2!\,11!}\binom{44}{1}$$

$$= 13\left[\binom{48}{5} - \frac{12!}{2!\,11!}\,44\right]$$

$$= 13\left[\frac{48!}{5!\,43!} - 6\cdot 44\right]$$

$$= 13\left[\frac{48! - 6!\,44!}{5!\,43!}\right]$$