Math 530

7.5 $\boxed{\text{example \#1}}$ (bot) $a_n = a_{n-1} + n$

(2) $\sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} (a_{n-1}x^n + nx^n)$

(3) $= x\sum_{n=1}^{\infty} a_{n-1}x^{n-1} + \sum_{n=1}^{\infty} nx^n$

(4) $= x\sum_{m=0}^{\infty} a_m x^m + \boxed{\sum_{n=0}^{\infty} \binom{n}{1} x^n}$

(5) $= x\,g(x) + \dfrac{x}{(1-x)^2}$

$$\binom{k+n}{k} = \binom{k+n}{n}$$

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(2) $\sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} (a_{n-1}x^n + nx^n)$

$= [a_0x^1 + a_1x^2 + a_2x^3 + \dots] + [x + 2x^2 + 3x^3 + \dots]$

know: $g(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$

$\Rightarrow = x\underbrace{[a_0 + a_1x + a_2x^2 + \dots]}_{g(x)} + [x + 2x^2 + 3x^3 + \dots]$

$= xg(x) + [x + 2x^2 + 3x^3 + 4x^4 + \dots]$

know: $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \dots$

take $\dfrac{d}{dx}$

$$\boxed{\dfrac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \dots}$$

$$\hookrightarrow \; = xg(x) + x\,[\underbrace{1 + 2x + 3x^2 + 4x^3 + \cdots}_{\frac{1}{(1-x)^2}}]$$

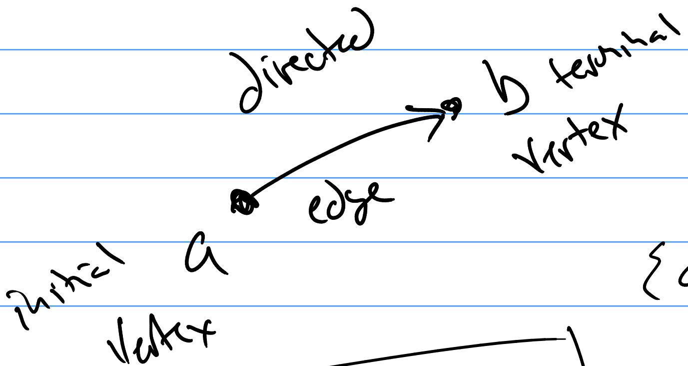
$$= x(g(x)) + x\left(\frac{1}{(1-x)^2}\right)$$

$$(5) \quad = \boxed{xg(x) + \frac{x}{(1-x)^2}}$$

Graph Theory

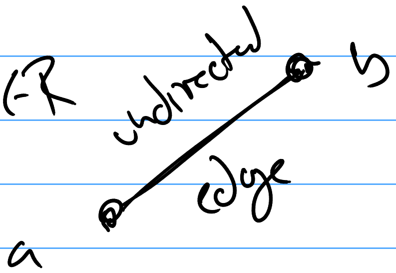
Chapter 1 Elements of Graph theory

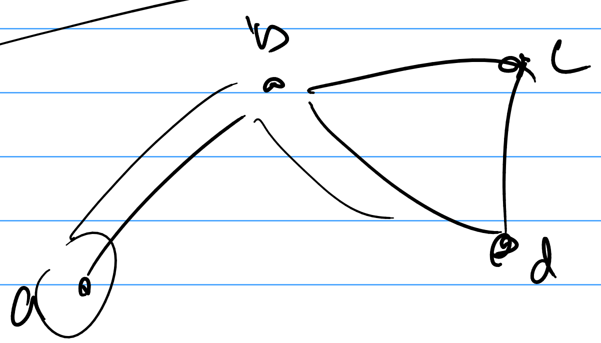
Represent Relationships between elements



$(a,b) \in R$

$\{a,b\} \in R$



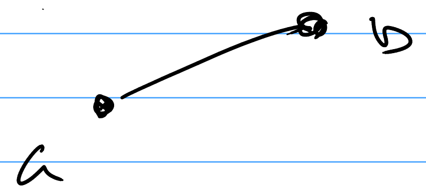


Elements: a graph $G = (V, E)$

V is a nonempty set of vertices
E is a set of $\boxed{\text{edges}}$

Undirected Graph

edge $= \{a, b\}$

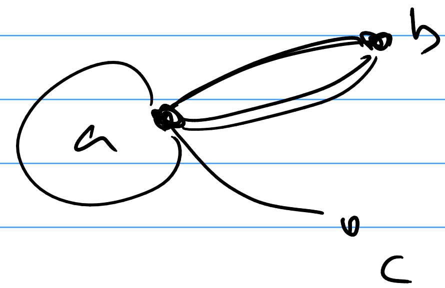


terms:

If edge $= \{a,b\}$
call a, b $\boxed{\text{adjacent}}$
call edge to be <u>incident</u> to a and b

degree of vertex is number of incident edges. Except loop is counted as 2.

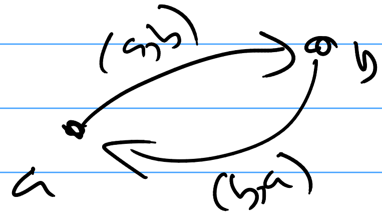
(ex)



$\deg(a) = 5$
$\deg(b) = 2$
$\deg(c) = 1$
$\overline{8}$

Directed Graph

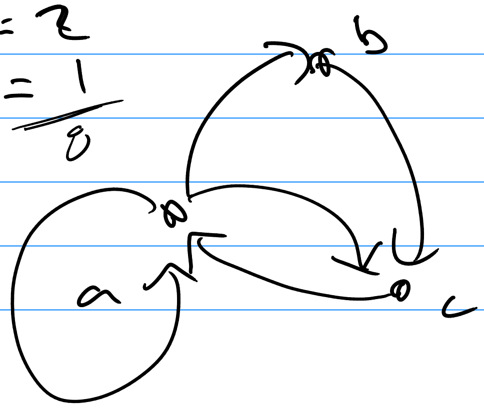
edge $= (a, b)$



if edge $= (a,b)$
call a adjacent to b.
call b adjacent from a.

indegree of a vertex is number of edges w/ vertex as terminal.

out degree of a vertex is number of edges w/ vertex as initial.



indegree	out degree
$\deg^-(a) = 2$	$\deg^+(a) = 3$
$\deg^-(b) = 1$	$\deg^+(b) = 1$
$\deg^-(c) = 2$	$\deg^+(c) = 1$
5	5

undirected graphs

$$\sum_{v\in V} \deg(v) = 2\,|E|$$

corollary: we must have an even number of vertices that have odd degree

directed graphs

$$\sum_{v\in V} \deg^-(v) = \sum_{v\in V} \deg^+(v) = |E|$$

Special Graphs

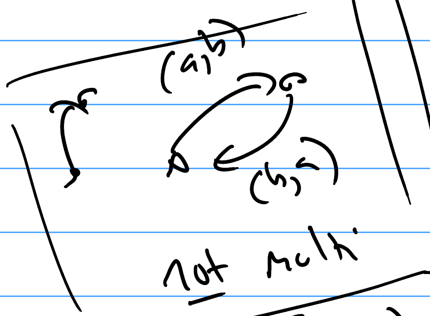
1st types

undirected

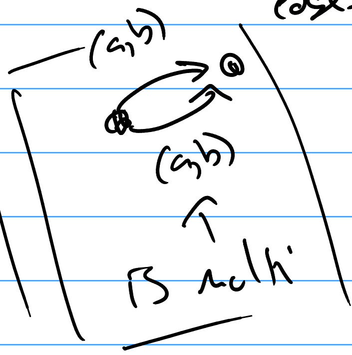
1. Simple → no loops, → no multiple edges
2. multigraph → no loops
3. pseudograph

directed ← loops are ok

1. simple directed graph → no multiple edges



not multi

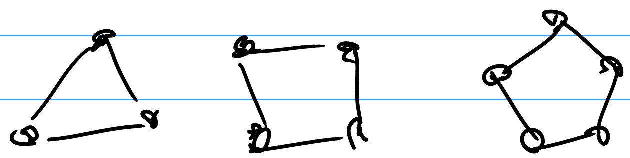


is multi

2. directed multigraph

2nd Special Simple Graphs

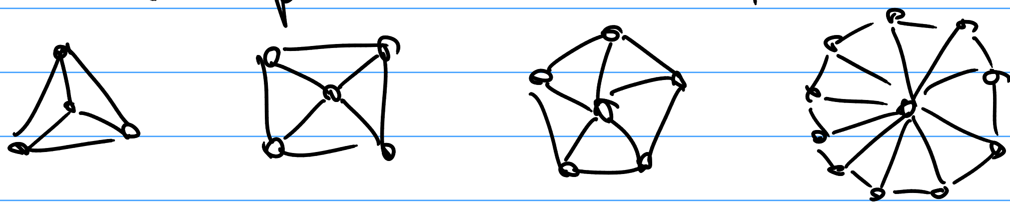
1. C_n



$n=3$ $n=4$ $n=5$

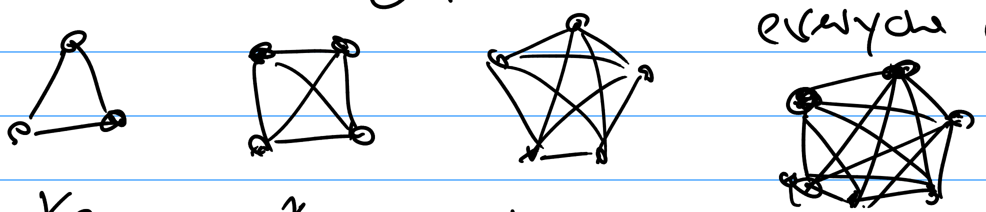
(2) W_n is C_n plus one extra vertex

W_n



$n=3$ $n=4$ $n=5$ $n=10$

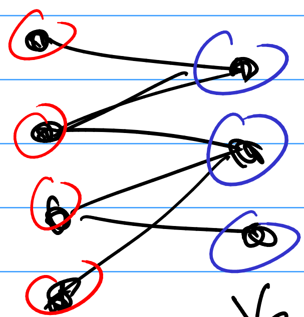
(3) K_n Complete graph (everyone connects to everyone else)



K_3 K_4 K_5 K_6 ...

(4) **Bipartite** means $V = \underbrace{V_1 \cup V_2}_{\text{partition}}$,

and all edges only connect between V_1 vertices to/from V_2 vertices



V_1 V_2

test? coloring th^m

color vertices so that adjacent vertices have different colors and only use two colors $\rightarrow$ bipartite

Graphs $G = (V, \boxed{E})$

Representations

(1) lists $G = (\underbrace{\{a, b, c\}}_{\boxed{V}}, \{(a,a), (a,b), (b,c), (c,c)\})$

(2) dictionary

$$\begin{array}{l} V \\ a : a, b \\ b : c \\ c : c \end{array}$$

(3) Adjacency Matrix

$$A_G = \begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 1 & 0 \\ b & 0 & 0 & 1 \\ c & 0 & 0 & 1 \end{array}$$

(4) Picture / Graph