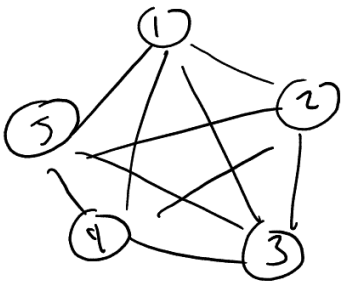
Math 530

Q5 (1.3 # 7)

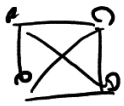
K_5



$|E| = 4+3+2+1$

$= \frac{4(5)}{2}$

$K_n \quad |E| = \frac{(n-1)n}{2}$

	K_1	K_2	K_3	K_4	K_5
	•				
$\|E\| = M$	0	1 $\xrightarrow{+2}$	3 $\xrightarrow{+3}$	6 $\xrightarrow{+4}$	10
	0	1	2+1	3+2+1	4+3+2+1
					$\frac{4(5)}{2}$

$\frac{(n-1)n}{2} = M$

$(n-1)n = 2M$

$n^2 - n = 2M$

$n^2 - n - 2M = 0$

$$n = \frac{1 \pm \sqrt{1+8M}}{2}$$

if $M = 3$

$n = \frac{1 \pm \sqrt{25}}{2}$

$n = \frac{1 \pm 5}{2}$

$n = \frac{6}{2} = 3$ $\quad \frac{-4}{2} = -2$ ✗

$$\boxed{n = \frac{1+\sqrt{1+8M}}{2}} \quad \text{vs} \quad n = \frac{1-\sqrt{1+8M}}{2}$$

$$\sum_{v \in V} \deg(v) = 2|E|$$

(1a) $|E| = 12$ $\deg(v) = 3$ always

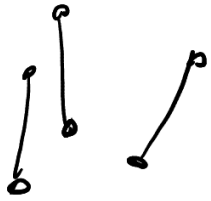
$$\sum_{v \in V} 3 = 2(12)$$

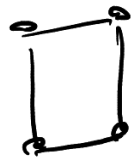
$3|V| = 24$ → $|V| = 8$

(1b) $3(4) + (|V|-3)(3) = 2(21)$ → $|V| = ?$

(1c) $|V|\,d = 2(24) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

$|V| = \frac{48}{d}$ $d = ?$





$d=1$ $|V| = 48$ $\deg = 1$ $\deg = 2$

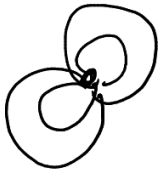
$d=2$ $|V| = 24$

$d = 3$ $|V| =$

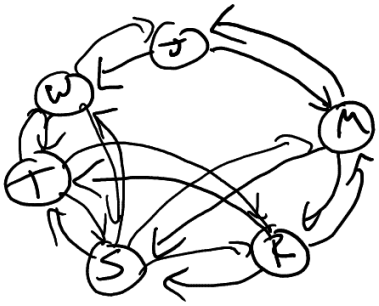
$d = 4$ $|V| =$

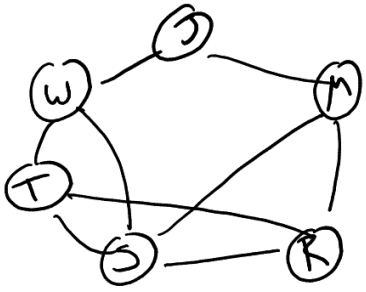
~~$d=5$~~ ~~$d=7$~~

$d = 6$



1.1 #4





(b) path from John to Steve? (shortest?)

J M R S T W

$A_G =$

	J	M	R	S	T	W
J	0	1	0	0	0	1
M	1	0	1	1	0	0
R	0	1	0	1	1	0
S	0	1	1	0	1	1
T	0	0	1	1	0	1
W	1	0	0	1	1	0

$A_G^2 =$ [rumor path after 2 days]

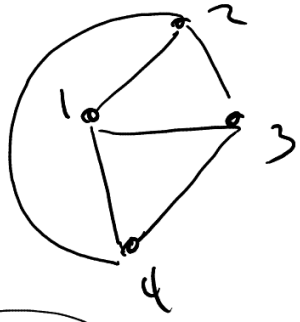
(c) John - Wendy? or John - Mary?

Planar Graphs and Maps

4 Regions

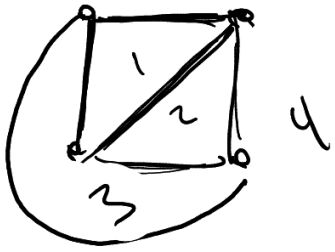
Degree of a region in maps = boundrys

dual graph

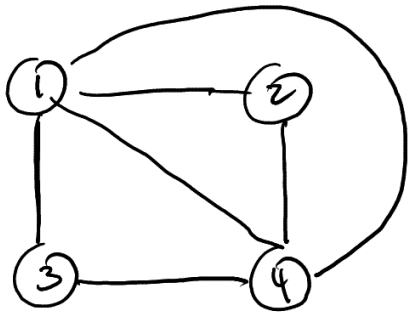


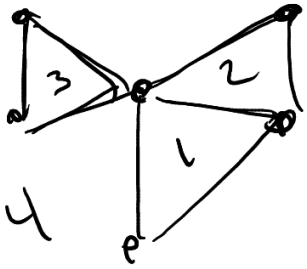
in dual region → Vertex

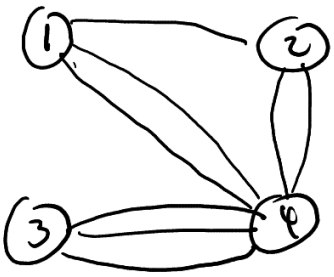
common boundry → edge

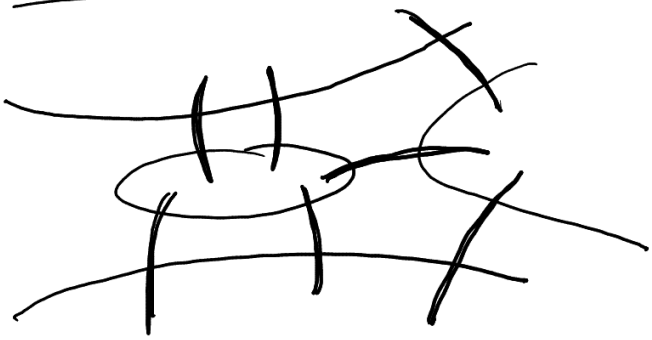


K_4

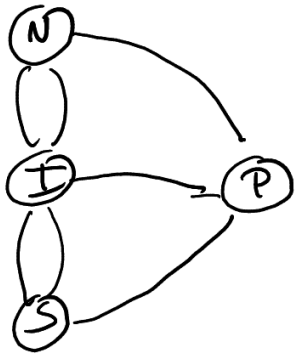








Bridge Town?



Path question?

① use all edges?

② use all edges once? (Euler trail)

③ use all edges once and begin = end? (Euler cycle)

Terms ① path = seq of edges

② simple path = seq of edges and edges occur once

(3) circuit = path begins/ends @ same spot

(4) trail is a simple path written using only vertex names.

(5) cycle is a simple circuit written w/ vertices.

Thm graph has an Euler cycle iff

$\deg(v)$ for all $v \in V$ is even

Thm graph has an Euler trail iff

$\deg(v)$ is odd for exactly two vertices

(And trail start/end at the odd deg vertices)
