Math 530

Q's

2.1 2

Note 1.4(1, 3bdf, 7acd)

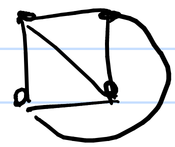
2.1(1, 2, 3, 9)

2.2(1, 3 5, 7a)

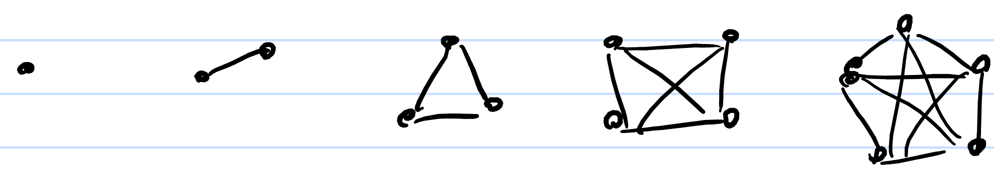
Euler circuits

(need all even degree)

Euler paths

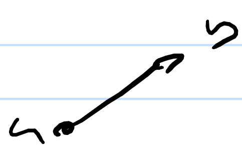
(exactly 2 of odd degree) also  K_4

K_n: $n=1$, $n=2$, $n=3$, $n=4$, $n=5$, ...



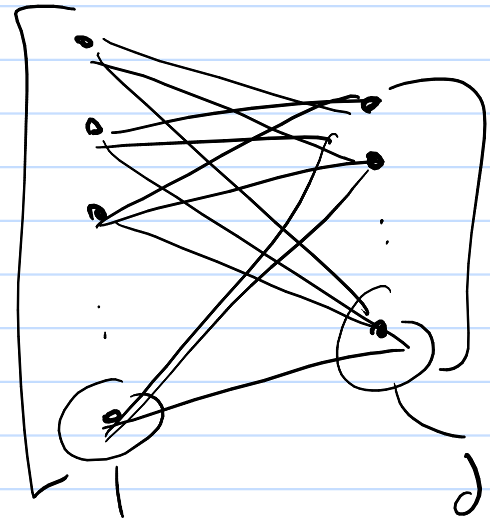
$\lvert V\rvert$	1	2	3	4	5	n
$\lvert E\rvert$	0	1	3	6	10	$\frac{n(n-1)}{2}$
deg(v)	0	1	$\boxed{2}$	3	$\boxed{4}$	$(n-1)$

2a) when deg(x) is all evens so $\lvert V\rvert = 3, 5, 7, \ldots$

2b) only K_2  it is a, b

2c) $K_{r,s}$

2c $K_{r,s}$



Euler cycle = all even

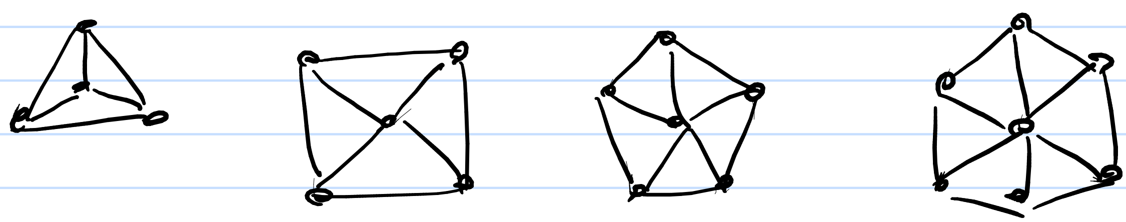
so r, s are even.

Euler path is

if one side is odd
the other has only $\underline{\underline{2}}$ vertices

$K_{2,\,odd}$ has Euler paths

(ex) W_n



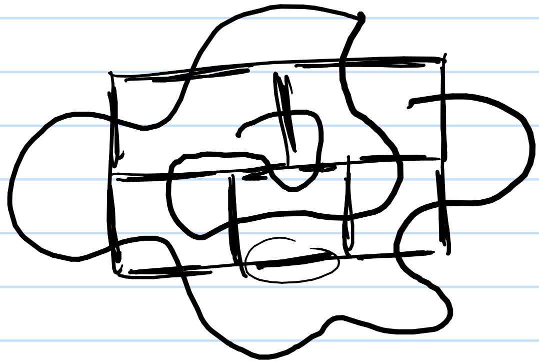
		n=3	n=4	n=5	n=6	n
|V|		4	5	6	7	(n+1)
|E|		6	8	10	12	2n
deg	axle	3	4	5	6	n
	outer	3	3	3	3	3

No Euler path

no Euler circuit

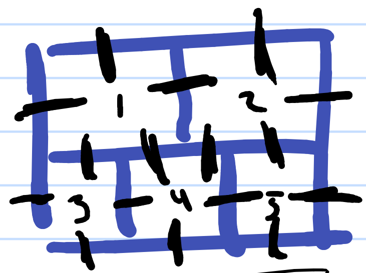
Apps of Euler Problems

①

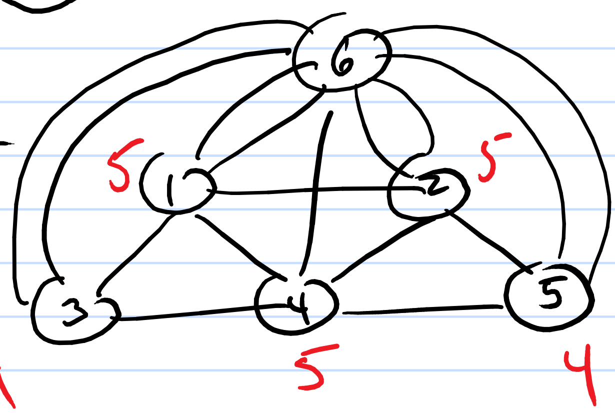


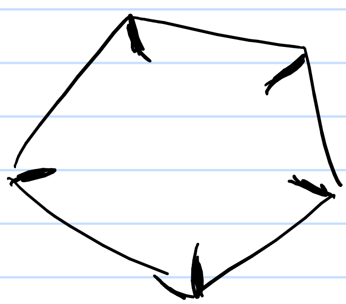
cut puzzle

6



dual of cut puzzle





deg of outside = 5

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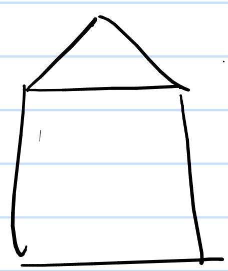
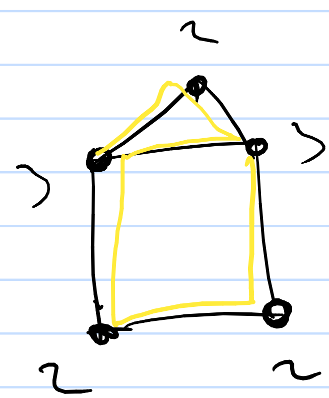
tracing

draw and do not stop

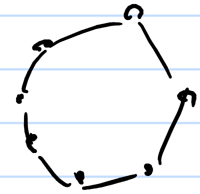




" Euler path or Euler circuit

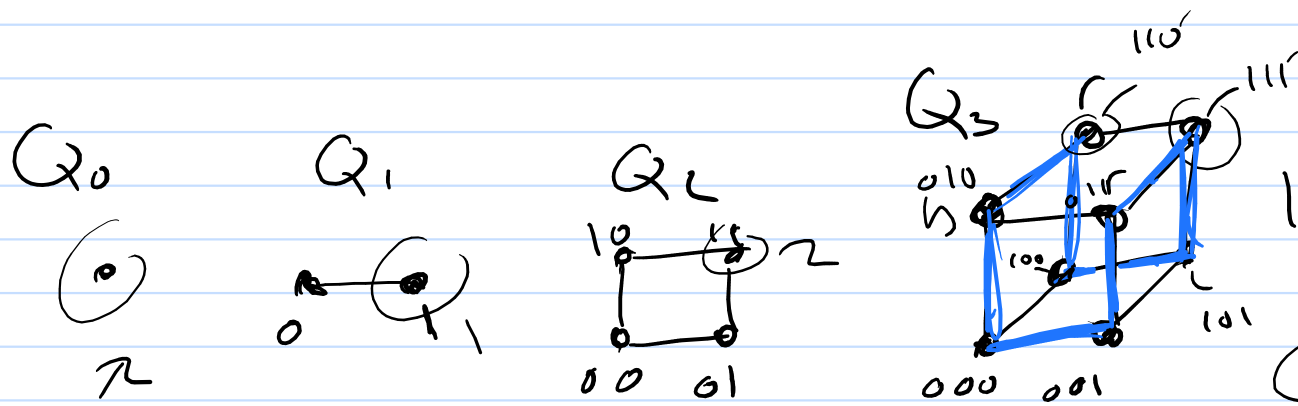
Hamilton:

C_n



Dirac if $\deg(v) \geq \frac{|V|}{2}$ then G has a Hamilton Circuit

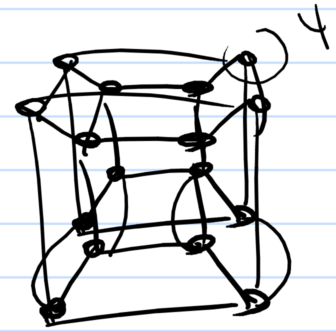
ORE's if $\deg(v_i) + \deg(v_j) \geq |V|$ with v_i, v_j are non-adjacent then G has a Hamilton Circuit

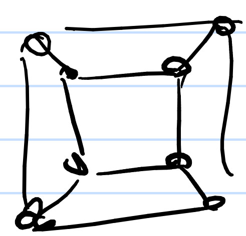


$|V| = 8$

000 = 0_1
001
010
011
100
101
110
111 = 0_8

Q_4

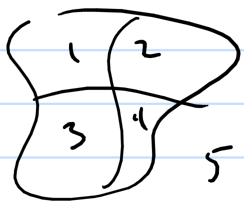




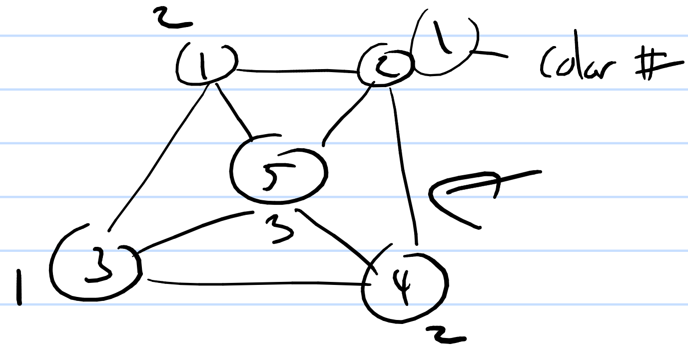
2.3 graph coloring

adj. vertices have different colors

(ex)



dual →



chromatic number of a graph is the absolute min. number of colors needed to color the graph

Note

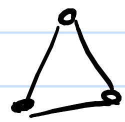
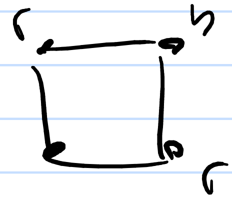
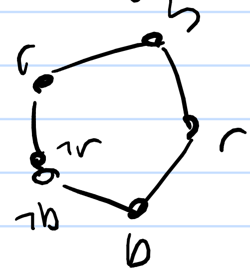
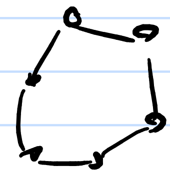
to show the chromatic number is K means to show $\boxed{\text{any}}$ K-1 coloring for a graph has two adj. vertices w/ same color.

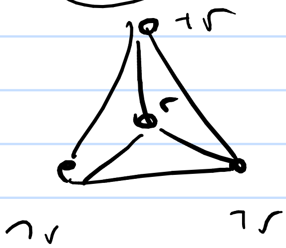
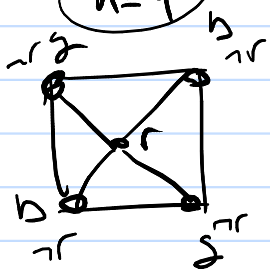
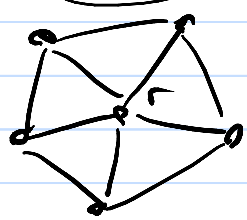
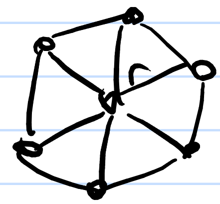
this is a NP-complete problem

Facts:

1. if G is planar, chromatic number is ≤ 4
2. K_n has a chromatic number of n.
3. bipartite has a chromatic number of 2.
4. if G has a K_i subgraph for i of its vertices then i is a lower bound for its chromatic number.

(ex) C_n

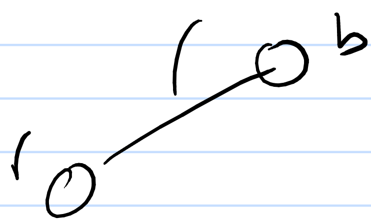
	$n=3$	$n=4$	$n=5$	$n=6$
				
chromatic number	3	2	3	2

W_n	$n=3$	$n=4$	$n=5$	$n=6$
				
chromatic	4	3	4	3

Application:

- **Scheduling**

conflict



chromatic number = number of isolation locations
